

## SYSTEM IDENTIFICATION FOR BASE ISOLATED BUILDINGS

Ary PAREDES<sup>1</sup>, Ruben BOROSCHEK<sup>2</sup>, Marcos ORCHARD<sup>3</sup>

### ABSTRACT

A group of procedures dealing with the nonlinear structural system identification for base isolated buildings are reviewed and compared herein. The study evaluates the effectiveness on addressing the nonlinear behavior of this type of systems, undergoing amplitude-varying earthquake motion, first through simulated acceleration responses from a set of numeric models including hysteretic representations accounting for the nonlinear behavior of the isolation system. Then, time history acceleration recordings of more than 90 seismic events that took place on an existing base isolated building in Chile, between 1993 and 2014, are used to validate the performance of the algorithms. Results show how the non-stationary behavior of the structure is detected by means of time-varying modal properties obtained from non-parametric (PSD), time-moving subspace (MOESP-MW) and recursive prediction error (RPEM-FF) techniques. It also points out Particle Filter (PF) as a promising model-based approach for the estimation of structural parameters, allowing to update a nonlinear finite element model (FEM) which considers the particular geometric and material features for any type of base isolated building.

*Keywords: Nonlinear System Identification; Base Isolated Structure; Time Varying Modal Parameters; Particle Filter; FEM Update*

### 1. INTRODUCTION

A base isolated building consists on a system that can be separated in two parts: the superstructure, which is typically a conventional building with relative high stiffness which is supported by a substructure, or isolation system, that provides flexibility and partially decouples the superstructure response from the ground motion. This feature produces a predominant response on the first horizontal mode and generates lower drifts between the superstructure floors compared to the ones produced on a fixed base building. System identification for base isolated buildings deals with the estimation of dynamic properties from structures showing localized nonlinear response under strong earthquake motions due to the isolation interface behavior. This article evaluate the effectiveness of two main groups of identification procedures on base isolated buildings. The first group identify varying modal parameters (frequency, damping and mode shapes) to capture the nonlinear behavior of the structure by means of a linear representation. For this purpose, algorithms based on the response frequency content, FFT - PSD (Bendat & Piersol, 1986), Multivariable Output-Error State Space (MOESP) with Moving Windows, MOESP - MW (Bakir, 2011), and a Recursive Prediction Error Method with Forgetting Factor, RPEM-FF (Safak, 1989), are used. On the other hand, structural parameters from a hysteretic model representing the nonlinear behavior of the structure, are identified performing a batch estimation by means of the Particle Filter PF algorithm. Simulated responses from a numeric base isolated building model subjected to different nonlinearity conditions, and subsequently a large number of time history acceleration data recorded on an existing base isolated building located in Chile, are used to test the identification procedures. Identified dynamic properties and performance of each method against an increasing nonlinearity are compared and discussed through the paper.

---

<sup>1</sup>Mr., Department of Civil Engineering, University of Chile, Santiago, Chile, [aparedes@ing.uchile.cl](mailto:aparedes@ing.uchile.cl)

<sup>2</sup>Dr., Department of Civil Engineering, University of Chile, Santiago, Chile, [rborosch@ing.uchile.cl](mailto:rborosch@ing.uchile.cl)

<sup>3</sup>Dr., Department of Electrical Engineering, University of Chile, Santiago, Chile, [morchard@ing.uchile.cl](mailto:morchard@ing.uchile.cl)

## 2. IDENTIFICATION ALGORITHMS

### 2.1 Fast Fourier Transform (FFT) – Power Spectral Density (PSD)

These two non-parametric identification algorithms (Bendat & Piersol, 1986) are based on the analysis of the signal frequency content. Peak peaking is used to find a predominant frequency of the system response. For the purpose of this paper, this approach is used to obtain a first approximation of frequencies and mode shapes of the system, with the latter based on the estimation of the frequency response function (FRF) for each time-history acceleration response.

### 2.2 Multivariate Output Error State Space algorithm with moving window (MOESP-MW)

It consists on a parametric subspace identification method (Bakir, 2011), which assumes a deterministic discrete linear state space representation for the system (Equations 1 - 2), using Hankel matrices, aiming to find the eigenvalues and eigenvectors of **A** and the **C** matrix, which are related to the modal properties of the structure (frequencies, damping and mode shapes) corresponding to a linear structure.

$$x_{k+1} = \mathbf{A} x_k + \mathbf{B} u_k \quad (1)$$

$$y_k = \mathbf{C} x_k + \mathbf{D} u_k \quad (2)$$

A model order determination and separation between structural identified properties is done on each moving overlapped time window by means of stabilization diagrams. In order to capture nonlinearities the response is split in consecutive time windows.

### 2.3 Recursive Prediction Error Method (RPEM)

This algorithm (Ljung, 1999) estimate a sought parameter  $\theta$  based on its previous value and the difference between the predicted output value  $\hat{y}(t)$ , given by a polynomial model representing the system, and the measured observation  $y(t)$ , term that is weighted by a Kalman gain factor  $K(t)$ . It follows the general expression given by (Equation 3)

$$\theta(t) = \theta(t-1) + K(t)(y(t) - \hat{y}(t)) \quad (3)$$

The form of the gain factor depends on the chosen model and the estimation algorithm, which in this case consist in the forgetting factor  $\lambda$  algorithm. This approach diminish the importance of old measurements exponentially such that an observation that is  $\tau$  samples old carries a weight that is equal to  $\lambda^\tau$  times the weight of the most recent observation. For tracking a time variant parameter (LTV) a value  $\lambda < 1$  must be specified (usually values between 0.9 – 0.995 are suggested, depending on the sampling frequency), while a value of  $\lambda = 1$  allows to track time invariant parameters (LTI). The general form for the polynomial representation is the showed by Equation (4). The autoregressive (AR) polynomial  $A(q)$ , is related to the system dynamic behavior and modal properties (frequencies and damping) by means of its polynomial roots, while the rational polynomial  $C(q)/D(q)$  includes the part that cannot be explained by past input-output data, coming from the noise model ( $y(t) | e(t)$ ).

$$A(q)y(t) = \frac{B(q)}{F(q)}u(t-k) + \frac{C(q)}{D(q)}e(t) \quad (4)$$

The objective is to estimate the polynomial coefficients from the chosen model, to get the discrete system properties. The selection of a given model structure (i.e. selecting polynomial orders), is based

on well-known and standard criteria (typically AIC, FPE and BIC) accounting for the parsimony of the selected candidate, while model validation is performed comparing the measured and predicted model response as well as applying some statistical tests (Safak, 1989).

## 2.4 Particle Filter (PF)

This model-based approach (Candy, 2011) consider a stochastic discrete state space representation for the system, consisting on a process and observation equation (Equations 5-6). It assumes a “random” sought parameter  $X$  with a probability density function  $p(X)$ . The parameter estimation uses Bayesian framework, where a prediction of the observation using the prior distribution  $p(X)$  is performed, and then an update of the sought parameter is carried out, through a likelihood function  $p(Y|X)$ , taking into account the measurements, filtering the prior and getting a posterior distribution  $p(X|Y)$  that accounts for the observed data  $Y$ .

$$x_{k+1} = x_k + \varepsilon_k \quad (5)$$

$$y_{k+1} = h(x_{k+1}, u_{k+1}, v_{k+1}) \quad (6)$$

Particle Filter aims to estimate an arbitrary form for the sought pdf, regrouping particles (i.e. parameters values) with assigned weights, by means of Importance Sampling and Resampling techniques, assuming a known transition distribution  $p(x_{k+1}|x_k)$ , which is defined in this case by a random walk process with parameter search noise  $\varepsilon_k \sim N(0, \sigma_\varepsilon)$  (Liu & West, 2001). This approach allows to address a “black-box” nonlinear system representation  $h(\cdot)$ , in order to estimate an arbitrary sought parameter distribution on a discrete form.

## 3. PART I: ANALYTICAL CASE STUDY

### 3.1 Numeric Models

A 10×8m in plan base isolated 4-story model constructed by the software SAP2000 is used to get simulated acceleration responses to external white noise seismic excitation with varying amplitude. Link elements with non-degrading hysteretic Bouc-Wen are used to represent the isolation characteristics. A total weight of 160 [Tonf] equally divided on each floor deck is assigned in the model. The isolation, material and geometric properties were chosen to match a value of 2.48 Hz and 2.78 Hz for the 1<sup>st</sup> mode frequency in Y and X directions respectively, as well as a damping of 5% for each mode. A post yield stiffness of 5% of the initial stiffness was assigned and three cases were considered in order to generate different hysteretic behaviors along with an increasing excitation amplitude (Figure 1). The fundamental frequency calculated by means of modal analysis considering the post yield stiffness as the effective stiffness of the isolators was equal to 0.83 Hz.

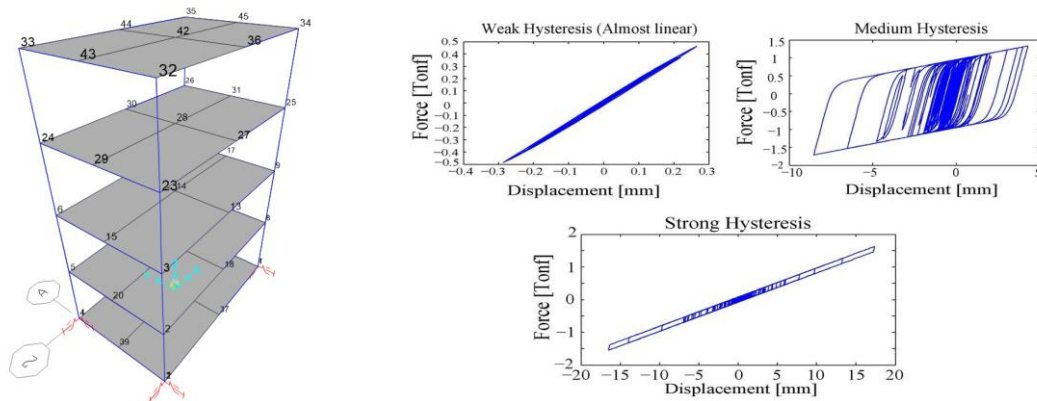


Figure 1. Numerical structure model and hysteretic cases for the analytical study

### 3.2 Identification Results

Non parametric techniques as FFT/PSD were not able to properly identify the nonlinearity of the system (Figure 2), confirming the results of Martinez et al [7]. Only a noticeable change for the 1<sup>st</sup> mode frequency and mode shapes on the weak/medium and strong hysteretic cases could be detected by these procedures and also by RPEM and MOESP (Figure. 3). This is presumably because of the selected non-degrading hysteretic bilinear model, which limits the change on the identified values to a transition between the modal properties related to the initial and post yield stiffness on the isolators. 1<sup>st</sup> mode damping estimation only showed an unstable increasing tendency on its values with nonlinearity.

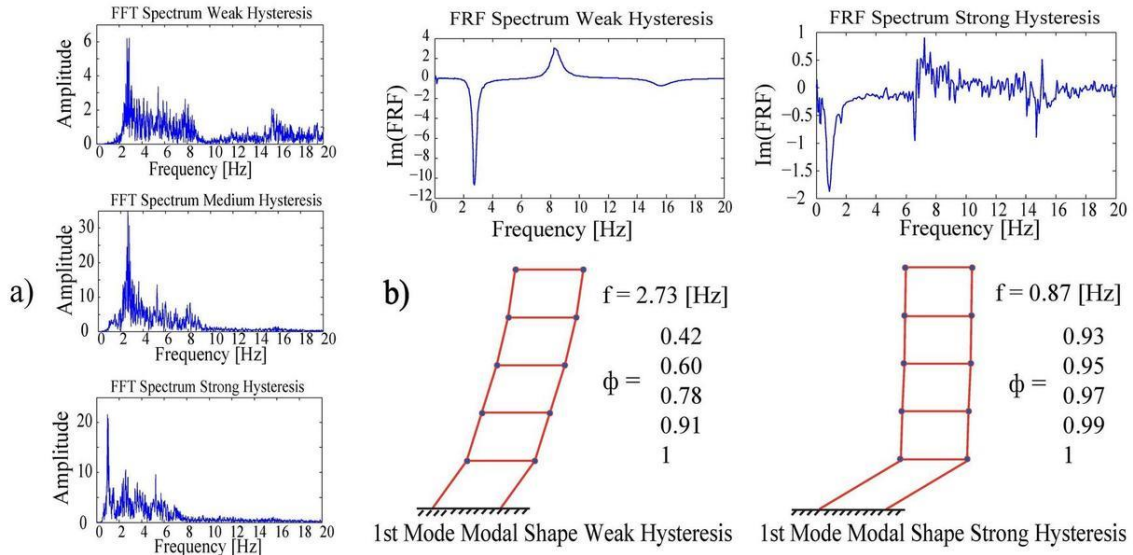


Figure 2. a) Change in FFT spectrum with hysteresis b) Change in FRF with hysteresis and 1<sup>st</sup> modal shapes

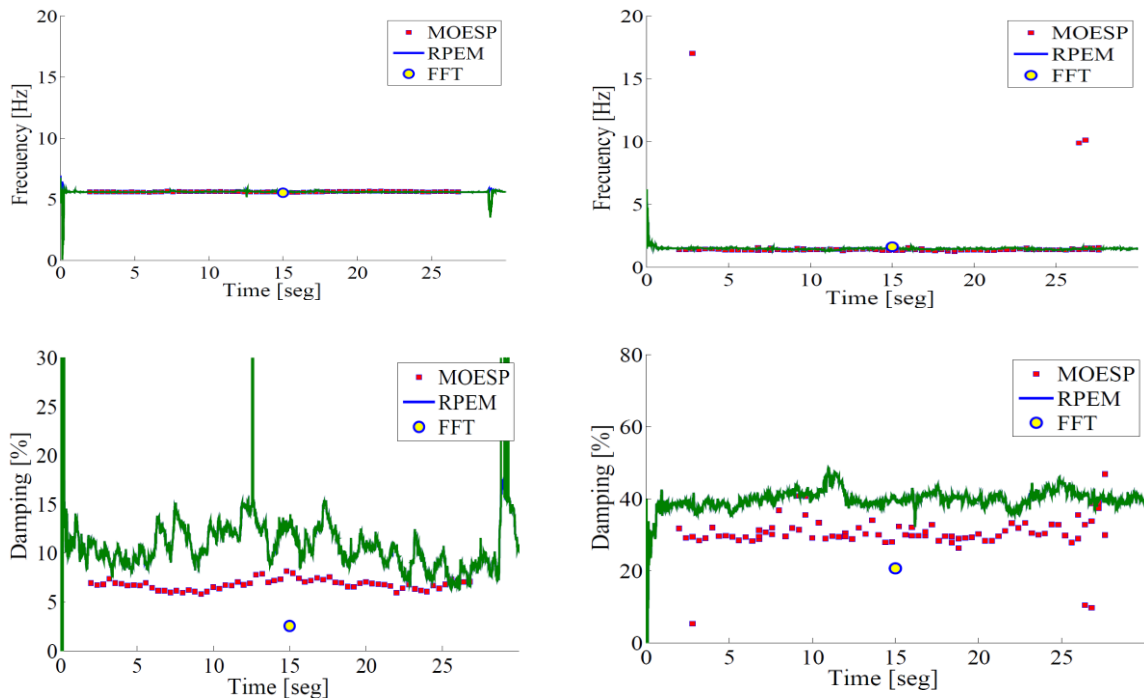


Figure 3. First mode identified properties with excitation amplitude: medium (left) and strong (right)

On the other hand, Particle Filter provides fairly good estimations of the structural parameters ( $k_1$ ,  $\alpha$ ,  $F_y$  and  $n$ ) using both, a deduced gamma form  $\Gamma(N \cdot L/2, 2/L)$ , with  $N$  and  $L$  the number of data and channels used as observation respectively, and a sigmoidal form dependent on the best estimation as likelihood functions, identifying successfully the assumed parameters of the Bouc-Wen model representing the isolation behavior (Figure 4), allowing to reproduce its hysteretic response accurately (Figure 5). Different levels of non-linear behavior are characterized by a unique model, while the excitation is strong enough to produce a hysteretic state of the system (on a weaker excitation, only initial stiffness presents good estimates). Quality of defined initial conditions, noise level in measurements and the number of particles play an important role in the convergence effectiveness to the right properties, in order to avoid multimodal posterior distributions, which are likely to take place due to the nonlinearity of the model and a limited number of measurements (non-observability problem). Also it was observed that the incorporation of relative displacement as an observation of the system provides better performance on parameter estimation, allowing us to discard solutions from a multimodal identified posterior distribution.

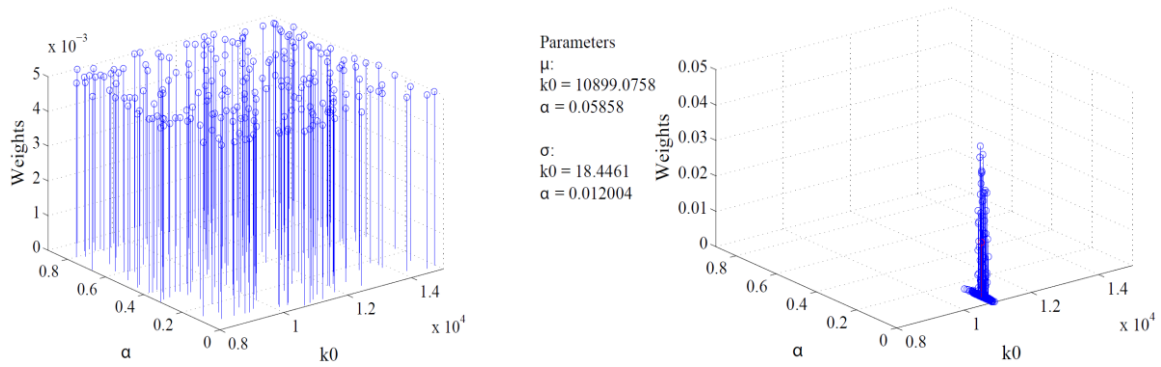


Figure 4 Estimated posterior distribution from identified parameters using Particle Filter (PF)

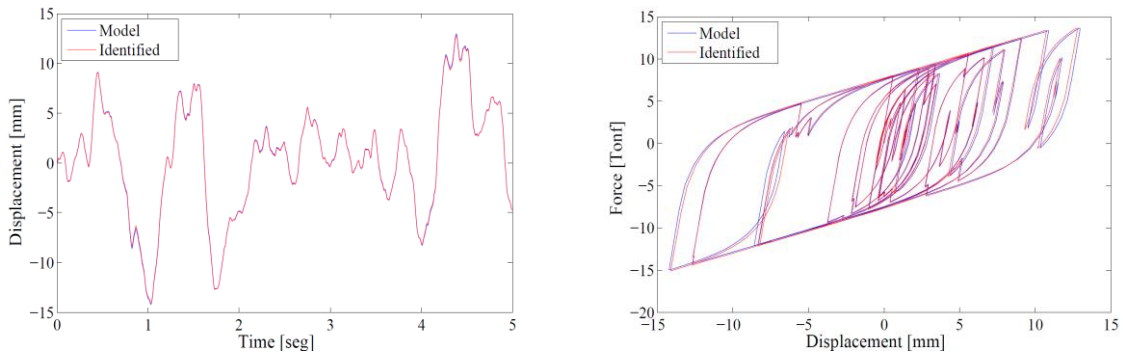


Figure 5. Displacement and hysteretic response estimation with identified parameters with PF.

## 4. PART II: IDENTIFICATION ON AN EXISTING BASE ISOLATED BUILDING

### 4.1 Studied Base Isolated Building

The based isolated building used for this study consist in a 4-story low cost housing project, supported on eight high damping rubber isolators (HDR), constructed in Santiago, Chile in 1992 [8]. It's been instrumented with a local network of digital accelerometers that has recorded more than 90 seismic events between 1993 and 2014, including a major earthquake taking place on February 27, 2010 with magnitude Mw 8.8. The first floor is composed of reinforced concrete and the upper three of confined masonry. All floors have a 10 cm thick reinforced concrete slab, with the wooden roof. The bearings, 31.5 cm in diameter and 32 cm high, were composed of 34 layers of 6.7 mm thick high damping rubber



and 33.2 mm steel shims. The building is instrumented with 2 triaxial accelerometers, in the first (L) and fourth floor (C). The seismic excitation is measured by another accelerometer located in the foundation level (F), recording acceleration in E-W, N-S and Vertical directions (Figure 6). The records were preprocessed by bandpass filtering with cutoff frequencies of 0.25 Hz and 30 Hz.

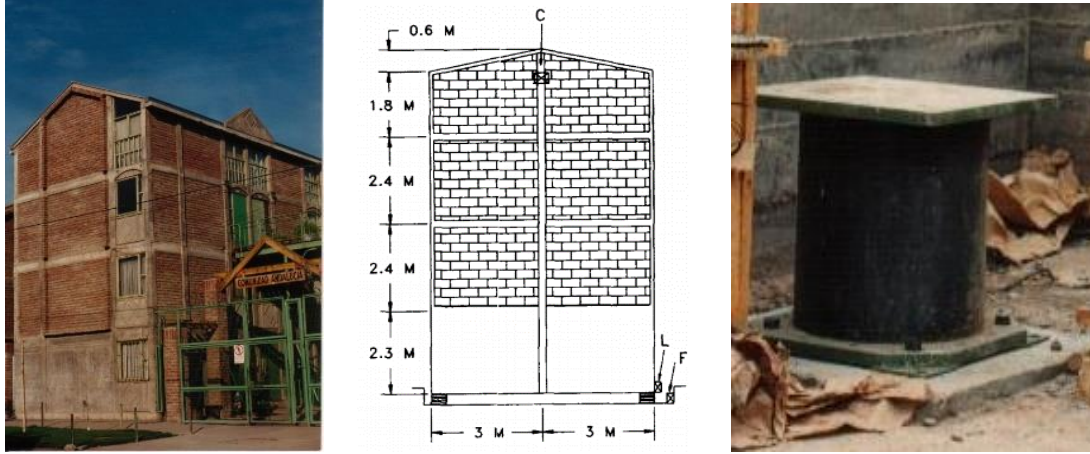


Figure 6. Andalusia Base Isolated Building with HDR isolation

#### 4.2 Identification Results

Two types of behavior depending on the relative displacement in the isolation interface are observed. 1st mode frequency values vary from 0.75 Hz to 5.59 Hz in the N-S direction, and 0.64 Hz to 6.07 Hz for the E-W direction for seismic events with a PGA ranging from 0.001 to 0.33 [g] and relative displacements between 0.03 to 10.6 cm. Higher frequency values obtained for events with lower PGA agrees very well with environmental vibration test performed previously on the building (Figure 7). Unlike the analytical case, identified frequencies display a smoother transition respect to displacement at the isolation interface level, because of the nonlinear and coupling behavior of the isolators, a feature that was not addressed on the analytical study (Figure 8). Vertical mode frequencies about 15-16 Hz for almost every seismic event are identified. Identified modes are coupled within orthogonal directions.

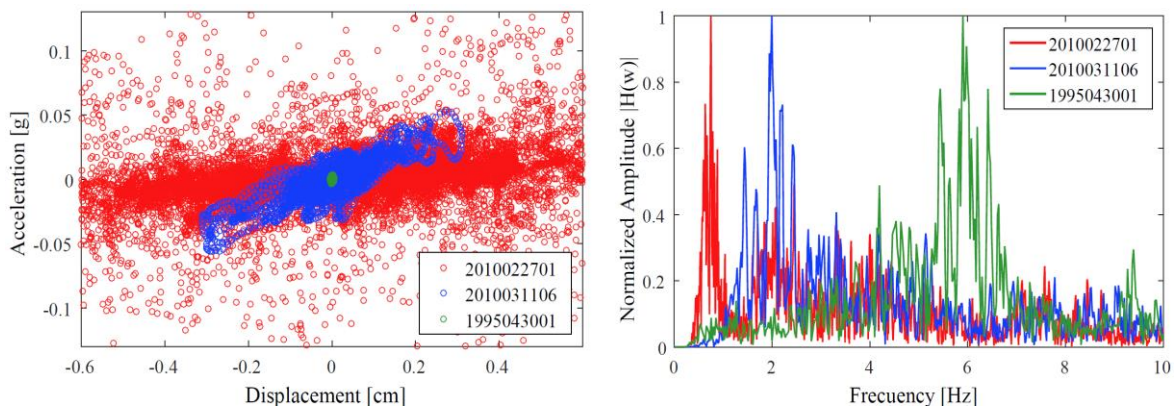


Figure 7. Estimation of hysteretic response and FFT for 3 different levels of excitation on seismic events

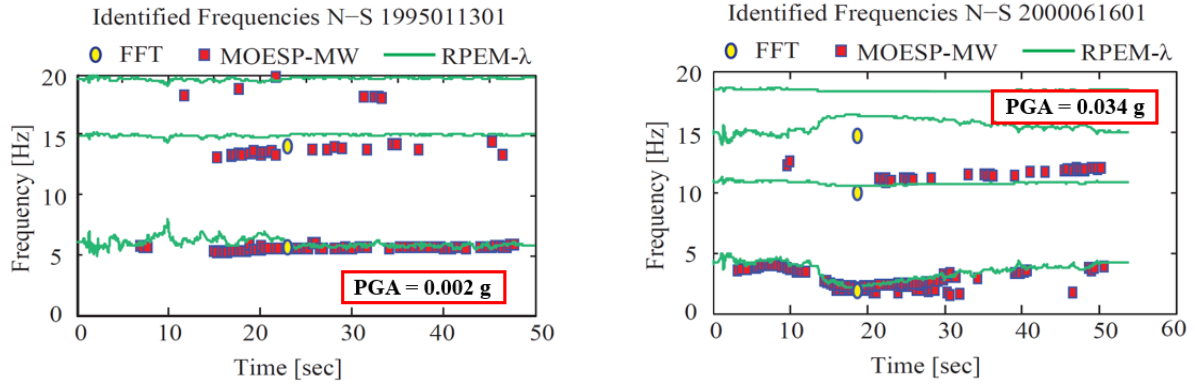


Figure 8. Identified frequencies for lower (left) and higher (right) PGA, by RPEM and MOESP-MW

For Particle Filter, acceleration measurements were used in order to estimate the isolator properties. Consistent structural parameters compared to values obtained from pullback experimental tests performed on the building (Moroni, Sarrazin & Boroschek, 1998) were estimated, for an assumed nonlinear model aiming to incorporate the coupled behavior of the existing isolated building. The estimates for the sought parameters were:  $k_{isol} \sim 50 - 80$  Tonf/m,  $F_y \sim 0.5 - 0.8$  Tonf,  $\alpha \sim 0.08 - 0.2$  and  $n \sim 0.2 - 0.5$  (exponent defining Bouc-Wen hysteretic model for isolator resisting force). Contribution of higher frequencies was not able to be addressed, underestimating the peak responses. Emphasis must be put on the selection of good initial values, especially for the initial stiffness ( $10^2$  Tonf  $< k_{in} < 10^4$  Tonf) due to the variety of models that could adjust the measurements (multimodal parameter distributions). Displacement observations seemed to help for discriminating between multiple solutions given by a wide search range for admissible initial stiffness values identified using only the accelerations responses. Representation of the identified parameters as a probability distribution also allows getting information about the uncertainty of the estimates for different hysteretic levels. Limitations on nonlinear models aiming to fit the measured response forced to change the original likelihood function by a sigmoidal one dependent on the best estimate, allowing to address the gap between the assumed analytical model and the real structural behavior, being only possible to estimate the dynamic behavior of the structure partially, but with reasonable good results (Figure 9).

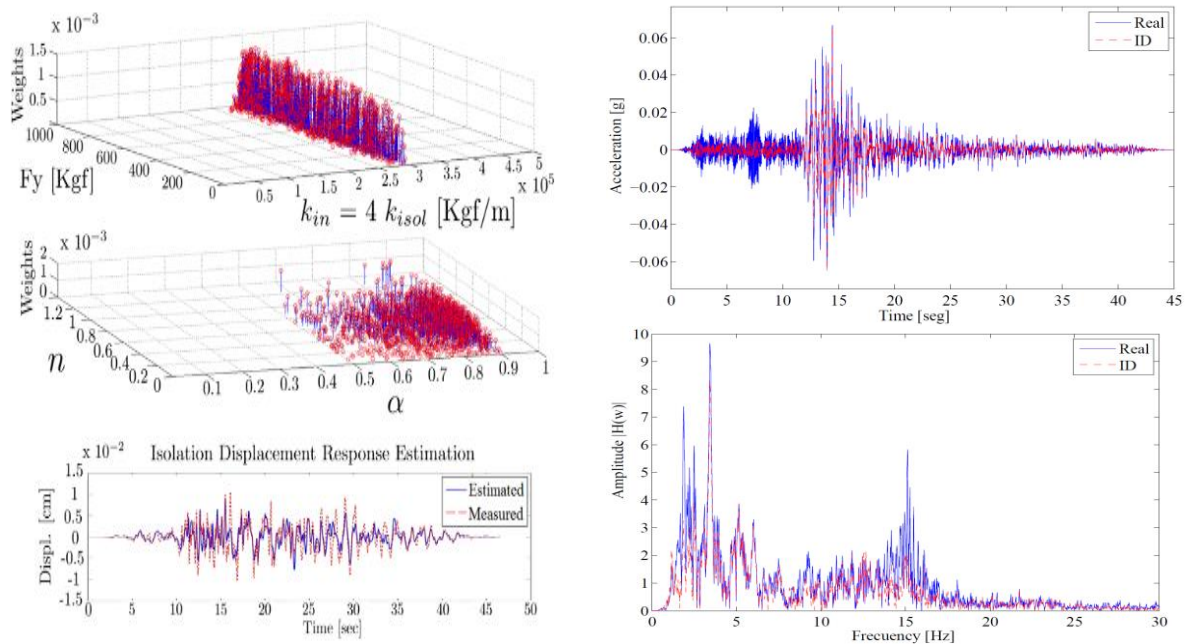


Figure 9. Parameter estimation by PF assuming 1D model (left) and response estimation using PF assuming 3D finite element model (right)

## 5. CONCLUSIONS

An analytical and experimental study on 4 identification techniques applied to base isolated buildings has been presented. FFT/PSD, MOESP-MW and RPEM identify nonlinear features of the system by means of varying modal properties, presenting strong limitations, mainly on damping estimation, showing only an increasing tendency with nonlinearity. Selection of non-degrading bilinear hysteretic model representing the isolator's behavior limits the performance of these methods, identifying the predominant stiffness related frequency for different levels of response nonlinearities. Mode shapes determination are only consistent for the horizontal first mode as nonlinearity increases and for vertical direction with a linear behavior. Both, RPEM and MOESP-MW are able to detect frequency changes discriminating responses corresponding to weak and strong phases, obtaining consistent values with previous identified frequencies on the existing base isolated building, presenting a practical use mainly for a frequency based real time monitoring applications on these type of systems.

Particle Filter showed advantage respect to the windows linear methods on allowing to select a single model for different levels of excitations, by means of a gamma-form likelihood function which allows to address the observation uncertainty, identifying in a more effective way the nonlinear hysteretic behavior of the structure. For this to be possible, the "clean" observation must correspond (or to be very similar in MSE terms) to the response generated by the assumed model for that particular excitation (or equivalently, is needed to assume an observation model that could get fairly good estimations for the system responses). Also, quality on the initial guesses for the sought parameters, number of particles and search space size play a fundamental role on reaching the right properties, particularly on finding a solution for a sought multimodal distribution case, where the uniqueness of the model is still a problem that depends on the model observability. On these cases, previous knowledge about the system, represented by the prior distribution and initial conditions are fundamental for reaching the true structural properties.

For the identification of the existing base isolated building, the simple bilinear representation for the hysteretic behavior of the isolators incorporated on the assumed models (either a planar or a 3D finite element model) could limit the identification performance for the Particle Filter, being necessary to incorporate a sigmoidal likelihood function dependent on the best estimation for the set of particles, addressing the gap between the assumed analytical governing equations and the real structural behavior. Consistent structural properties were obtained by this procedure for the cases where stronger excitations took place, while for weaker excitations, only initial stiffness presented good estimations. Better estimations are expected if a more robust model is included, but taking into account that if this implies an increase on the number of parameters to be estimated, given the fixed number of observations available to perform the identification, it could increase the multimodality of the sought distribution, making more difficult to find the corresponding solution without considering appropriately the initial knowledge about the system to be identified.

## 6. ACKNOWLEDGMENTS

The authors would like to thank Department of Civil Engineering University of Chile for providing the data and necessary information to develop the research.

## 7. REFERENCES

- Bendat, J. S., & Piersol, A. G. (1986). Random data: measurement and analysis procedures.
- Bakir, P. G. (2011). Automation of the stabilization diagrams for subspace based system identification. *Expert Systems with Applications*, 38(12), 14390-14397.
- Candy, J. V. (2011). Bayesian signal processing: Classical, modern and particle filtering methods. John Wiley & Sons.



Martínez J.M., R. Boroschek, J. Bilbao. "System Identification procedures for nonlinear response of Buckling Restrained Braces". 7th International Conference on Structural Health Monitoring of Intelligent Infrastructure, SHMII, Torino, Italy, 1-3 July 2015

Ljung, L. (1999). System identification (pp. 370-374). Birkhäuser Boston.

Liu, J. y West, M. (2001). Combined parameter and state estimation in simulation-based filtering. En Sequential Monte Carlo methods in practice, pp. 197{223. Springer.

Moroni, M. O., Sarrazin, M., & Boroschek, R. (1998). Experiments on a base-isolated building in Santiago, Chile. Engineering Structures.

Safak, E. (1989). Adaptive modeling, identification, and control of dynamic structural systems. I: Theory. Journal of Engineering Mechanics, 115(11), 2386-2405.