

# Modal parameter variations due to earthquakes of different intensities

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## 1 Nomenclature

- $\omega_j$  : Modal angular frequency. Mode j.  
 $f_j$  : Modal frequency. Mode j.  
 $\xi_j$  : Modal damping ratio. Mode j.  
 $ag_i(t)$  : Recorded base acceleration. Direction i  
 $L_{j,i}$  : Modal participation ratio for base acceleration  $ag_i(t)$ . Mode j  
 $\phi_{j,p}$  : Modal shape vector at position p. Mode j.  
 $y_j(t)$  : Modal response. Mode j.  
 $a_p(t)$  : Estimated acceleration of MIMO algorithm at position p  
 $E$  : MIMO goodness of fit error.  
 $\alpha_p$  : Weight coefficient at position p in MIMO goodness of fit error.  
 $RMS_{\text{modal},j}$  : Root Mean Square modal amplitude. Mode j  
 $\vec{a}_{X^{\text{th}} \text{ floor}}(t)$  : Measured acceleration, X<sup>th</sup> floor.  
 $\vec{\theta}_j$  : Unit vector defined by the mode shape's principal direction. Mode j  
 $[f_j - \Delta f_j, f_j + \Delta f_j]$  : Modal frequency confidence interval. Mode j.  
 $[X]_{[f_j - \Delta f_j, f_j + \Delta f_j]}$  : Band-pass filtered signal, between  $f_j - \Delta f_j$  and  $f_j + \Delta f_j$ .

## 2 ABSTRACT

This article presents the modal parameter variations during 55 low to high intensity earthquakes in a 22 story shear wall building located in Chile. Some of the earthquakes produced moderate damage in the structure. The building has been instrumented since 1997 with a network of 12 uniaxial accelerometers distributed on 4 different levels. Modal parameters are obtained through parametric Multiple Input - Multiple Output identification techniques. For the identified modal frequencies, decreasing values with respect to motion amplitudes are observed, in a range between 4% and 35% for the seismic records studied. For the modal damping ratios, increasing values with increasing motion amplitude are observed, with variation ranging from 50%, for the first translational modes, to over 120% for higher modes. For medium and low level earthquakes this variations disappear after the strong shaking has ended. This article also presents a preliminary analysis of the seismic records on the building during the Mw=8.8 earthquake occurred in Chile in 2010. Structural Damage was observed on the building and modal parameters changed significantly. During this seismic event, modal frequencies decreased up to 35% for the first translational modes. After the earthquake, modal frequencies had an 18% permanent decrease on average from initial values.

### 3 Introduction

Many studies on modal parameter variations of different buildings have been developed. For instance, Caltech's R.A. Millikan library has been monitored for this purpose, using accelerometers, since 1967. Studies like this have concluded that most of the variations observed in modal parameters, due to traffic, weather conditions (temperature, wind, rainfall, etc.), low to moderate intensity earthquakes, etc. do not necessarily implicate structural damage. In these cases, for example, modal periods have reported changes up to 3-4%, [1].

In particular, previous studies on the Chilean chamber of construction building, whose seismic records are analyzed in this research paper, have reported changes in the modal frequencies between 1% and 2% for ambient conditions, being temperature and rainfall the weather variables with the most environmental effect ([3]). As for the analysis of seismic records, a previous study on this building ([4]) found variations in modal frequencies up to 4 % during low to moderate intensity earthquakes, where none of which were reported to cause any structural damage.

### 4 Building properties

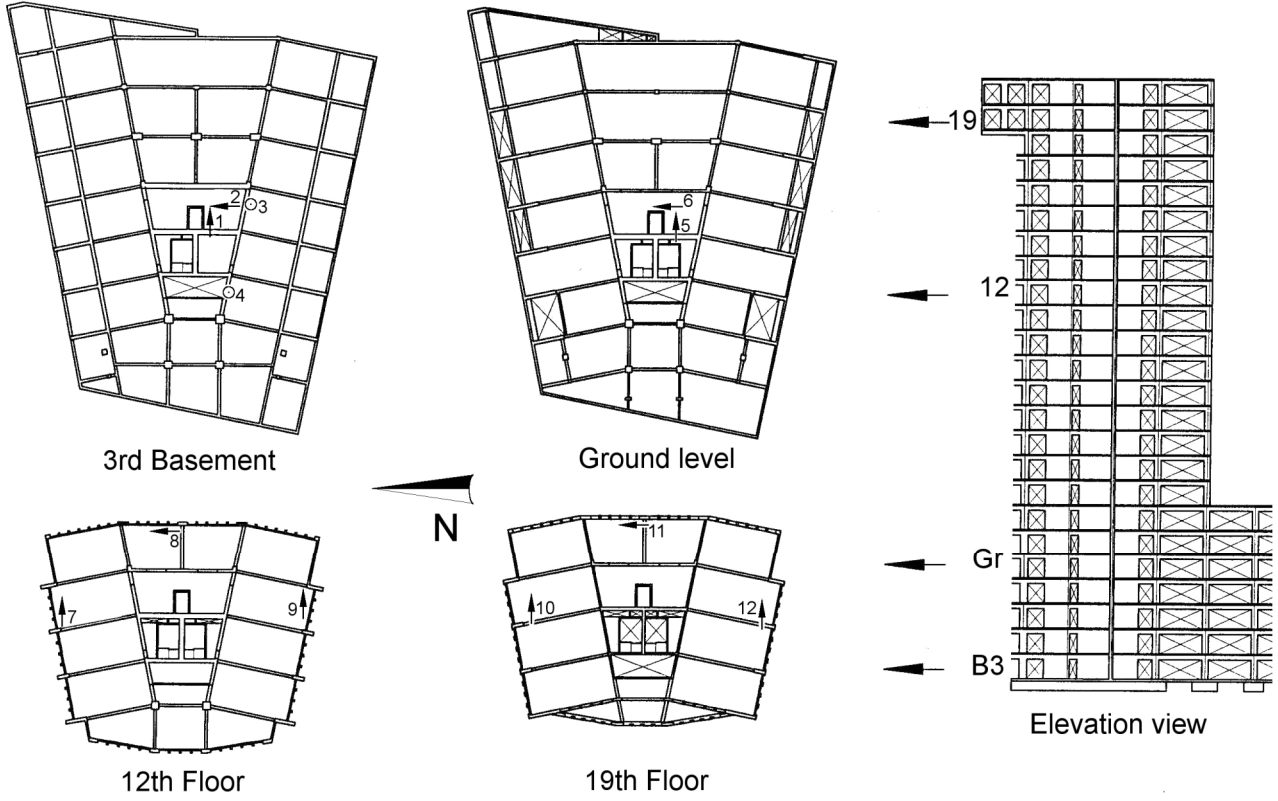
The Chilean Chamber of Construction Building, located in Santiago, Chile, is a 22 story, reinforced concrete, shear wall building. Since 1997, a continuous monitoring network has been recording both strong motion and ambient vibrations records of the building. The network consists of 12 uniaxial accelerometers located at four different levels of the structure, including its base, Fig.2.

As for its structural properties, the building is 85.5 meters high, and it is constituted of a wall-frame dual system, with most of the lateral loads resisted by walls. The walls are located mainly in the center, housing the stairs and elevator shafts. Small dimensions precast concrete wall panels are used on the perimeter. A common 1.50 m thick 305 square meter foundation slab supports the main structural walls and columns. Stories below ground level have a 300 mm thick perimeter wall, which resists both earth pressure and seismic loads. Wall area to plan area ratio is 6.3% for stories below ground and 4.0% for stories above ground. The floor slabs are 150 mm thick in all above ground stories and 170 mm thick below ground level. Soil foundation can be described as dense gravel, with a capacity of 1MPa for dynamic loads. Total floor area is 28,595 square meters, [7].

Since its installation, the structural health monitoring network has recorded several low and high intensity earthquakes. In this article, 55 of those seismic records, including the one corresponding to the recent M=8.8 earthquake occurred in Chile, are analyzed to determine modal parameter variations of the structure.



**Fig.1** Chilean chamber of construction building. External views



**Fig.2** Sensor Location. Chilean chamber of construction building

## 5 System Identification Technique.

Modal parameters for the analyzed seismic records were obtained through a parametric Multiple Input – Multiple Output (MIMO. [2], [9] and [10]) identification algorithm developed during this study. Based mainly on the dynamic modal equilibrium equations ((1), (2)), this identification algorithm searches for the optimal combination of parameters that best fits the measured response of the structure. Because the number of variables and the complexity of the target function, (3), the identification algorithm requires of adequate initial and limit values for all modal parameters to identify (modal frequencies, damping ratios and mode shapes).

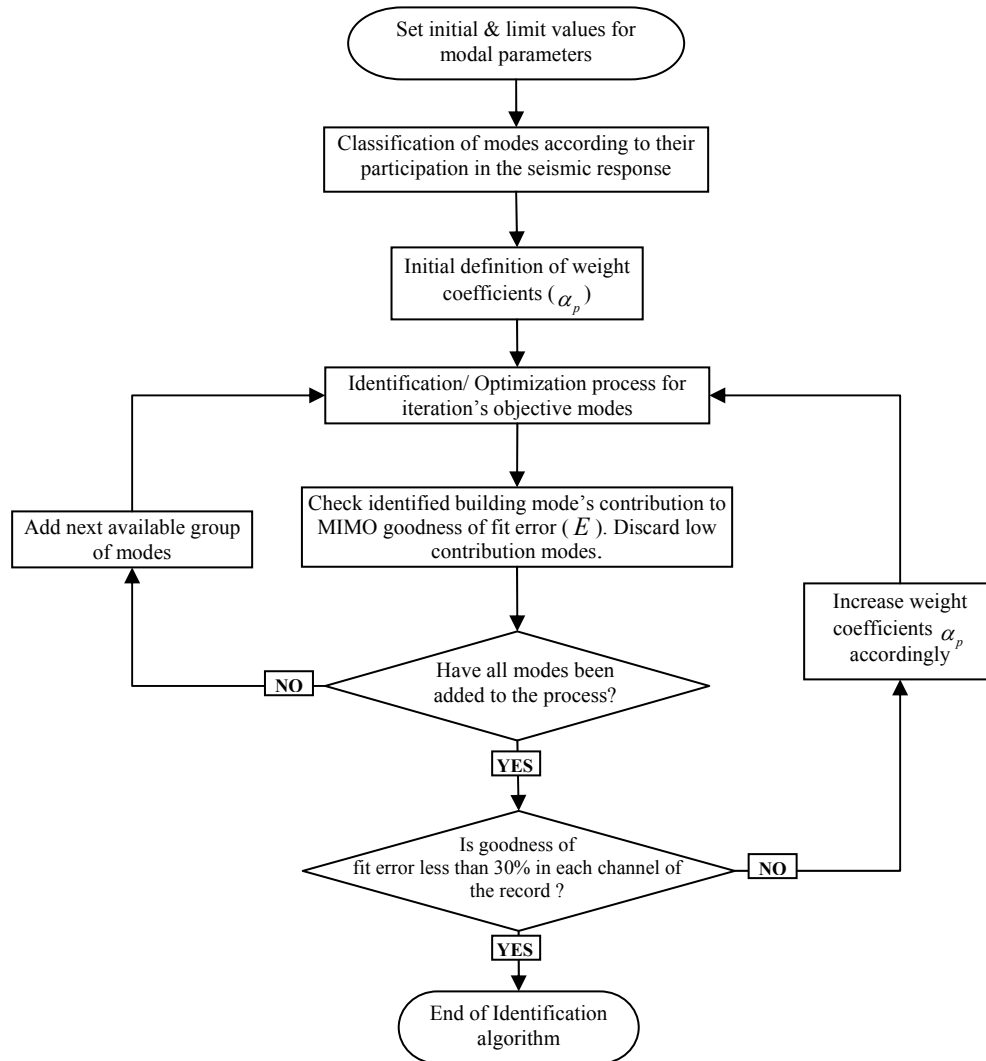
$$\ddot{y}_j(t) + 2 \cdot \omega_j \cdot \xi_j \cdot \dot{y}_j(t) + \omega_j^2 \cdot y_j(t) = \sum_{i=1}^k L_{j,i} \cdot a_{g,i}(t) \quad (1)$$

$$a_p(t) = \sum_{j=1}^N \phi_{j,p} \cdot \ddot{y}_j(t) \quad (2)$$

The target function of the optimization problem,  $E$ , is a weighted least-square-error between  $a_{0,p}(t)$  and  $a_p(t)$ . Weight coefficients  $\alpha_p$  control the importance of each channel of the response record in the modal identification process. Thanks to the normalization component  $\left( \sum_p \alpha_p \cdot \sum_t (a_{0,p}(t))^2 \right)$ ,  $E$  can be expressed in percentage units.

$$E = \sqrt{\frac{\sum_p \alpha_p \cdot \sum_t (a_{0,p}(t) - a_p(t))^2}{\sum_p \alpha_p \cdot \sum_t (a_{0,p}(t))^2}} \quad (3)$$

The Identification algorithm developed during this study reaches optimal modal parameters through an iterative process. First, building modes are classified according to their participation in the dynamic response during seismic events. Then, the iterative algorithm starts the identification process using only the modes with the highest response participation. At each iteration, a new group of modes is added to the process. Once all modes have been processed, only the ones with the most contribution to reduce the MIMO goodness of fit error ( $E$ ) are kept. In a final step, the error  $E$  for each channel of the response record is calculated, if one of them shows an error higher than 30%, its corresponding weight coefficient  $\alpha_p$  is increased accordingly and the process starts over. Fig.3 shows an explanatory flowchart of the Identification algorithm.

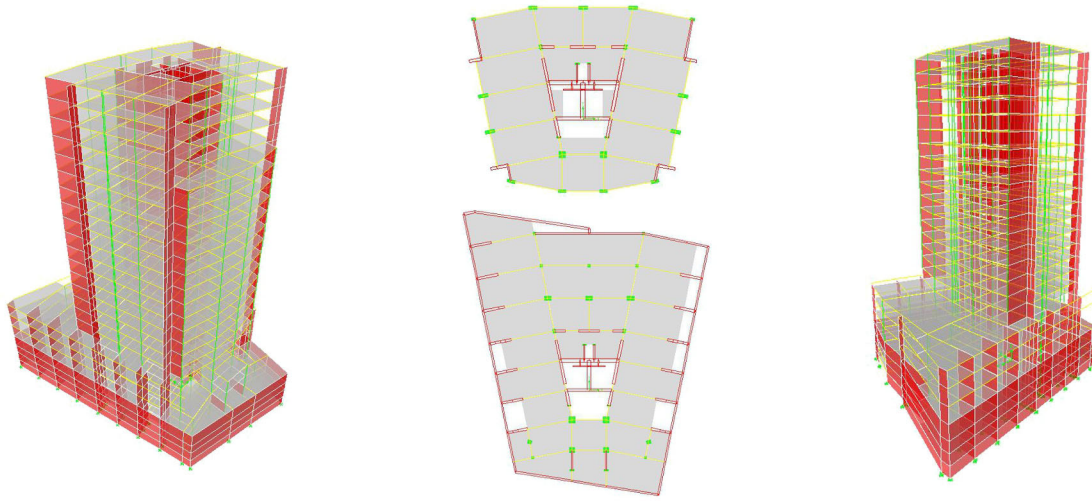


**Fig.3** MIMO Identification Algorithm

## 5.1 Numerical Validation

A finite element model of the building was developed to verify the results of the previously described identification algorithm. The properties considered in the model are the following: Concrete modulus of elasticity of 38000 [MPa]; fix foundation; element stiffness calculated from gross sections; rigid diaphragms assigned on each floor; concrete density of 2500 [kg/m<sup>3</sup>]; dead loads equal to 1.5 times self-weight; Live Load of 250 [kg/m<sup>2</sup>] on above ground floors and 500 [kg/m<sup>2</sup>] on underground floors; seismic weight calculated as *100% of Dead load plus 25% the Live Load*.

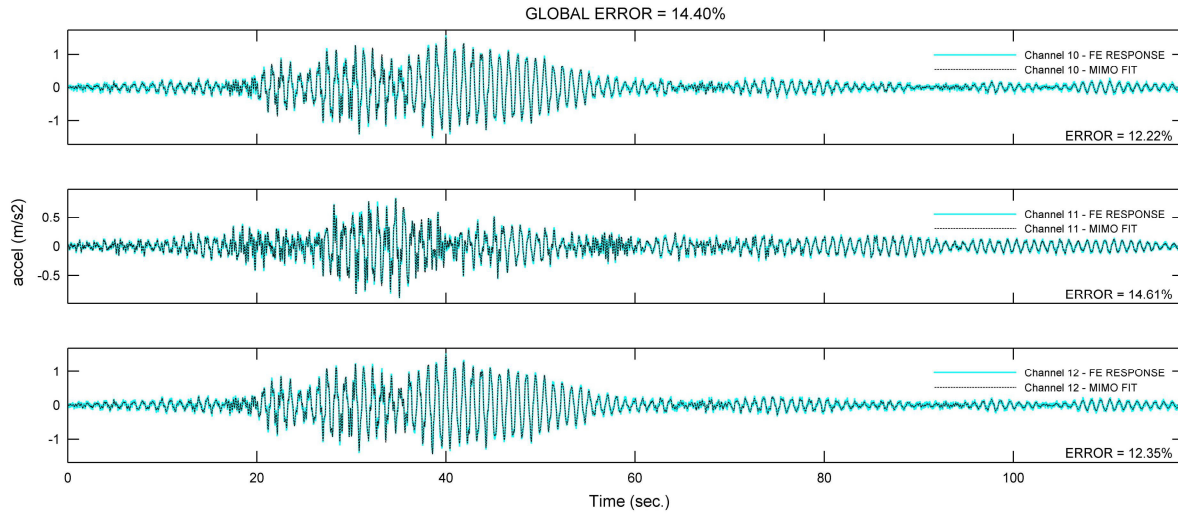




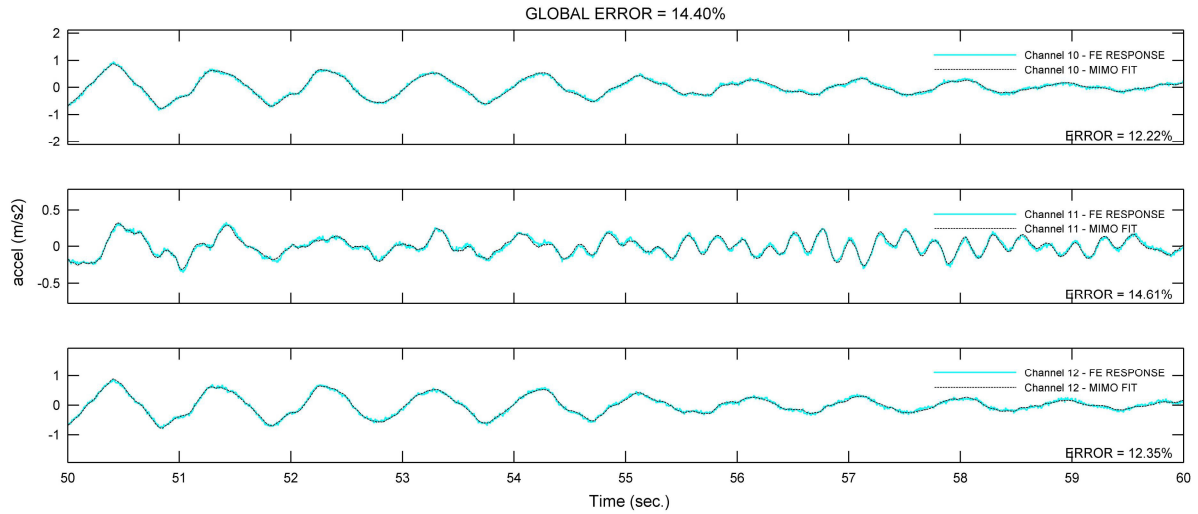
**Fig.4** Finite element model of the building. 3D and plan views

From this computer model, several analyses were performed using acceleration time-series recorded at the base of the structure during seismic events. Such base motion records (Input) and the resulting response of the computer model (Output) were used to validate the MIMO Identification algorithm. For better resemblance to real conditions, the chosen output signals on the computer model match the sensor location on the real monitoring network. In addition, 10% of the record amplitude was added to the response records as random Gaussian noise before the identification process.

As an initial indicator of the good results of the identification process, the goodness of fit error ( $E$ ) for these analyses ranged from 7% to 16%. Fig.5 and Fig.6 display an example of both the original contaminated time-series and the estimated fit. These figures show clearly how small the fit error is for the range of normalized error,  $E$ .



**Fig.5** Validation of analytical procedure. Computer model's contaminated record vs. Estimation by identified modal parameters. 19<sup>th</sup> Floor



**Fig.6** Validation of the analytical procedure. Computer model's contaminated record vs. Estimation by identified modal parameters. 19<sup>th</sup> Floor. Zoom

Modal identification over all analytical records with added 10% noise resulted in identified modal frequencies with less than 0.2% error. As for identified modal damping ratios, the error was less than 6%.

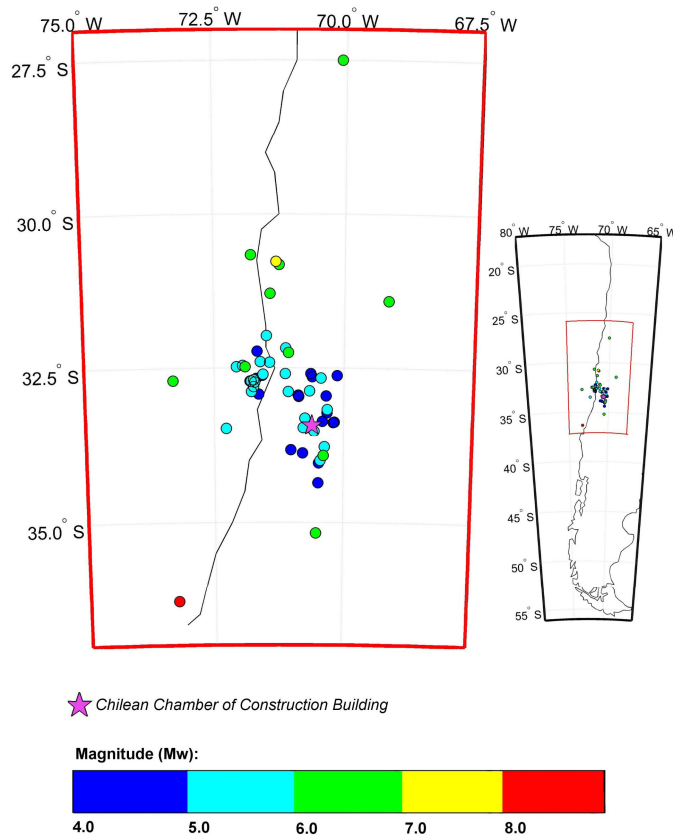
Table 1 shows the results of the identification process for one time-history analysis; these results are compared to expected values from the FE computer model.

**Table 1** Modal parameters: FE computer model vs. MIMO identification algorithm

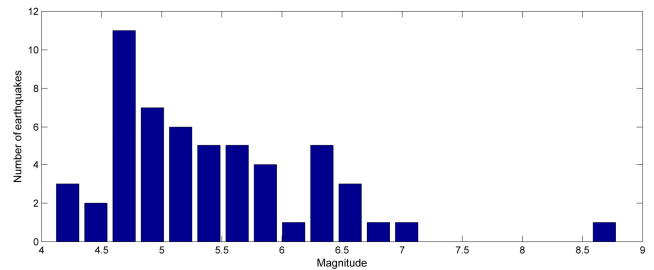
FE computer model		Identified by MIMO		Relative Difference (%)	
Freq [Hz.]	$\xi$ [%]	Freq [Hz.]	$\xi$ [%]	Freq	$\xi$
1.05	1.00	1.05	1.00	0.01%	0.26%
1.09	1.30	1.09	1.28	0.01%	1.27%
1.46	0.50	1.46	0.52	0.01%	3.58%
3.94	3.00	3.94	2.87	0.01%	4.36%
4.05	2.40	4.05	2.47	0.18%	3.04%
4.18	0.80	4.18	0.83	0.02%	3.86%
6.85	1.00	6.84	0.96	0.06%	4.45%
8.10	3.00	8.10	2.94	0.05%	1.89%

## 6 Description of the seismic records

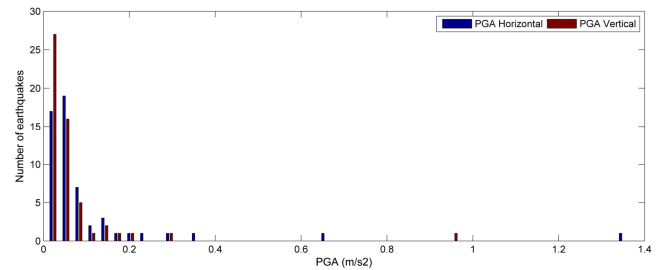
As mentioned before, this study involves the analysis of a total of 55 seismic records, obtained since 1997. The magnitude of the corresponding seismic events ranges between 4.2 and 8.8 (Mw). In addition, peak ground accelerations (PGA) at the base of the building ranged between 0.001g and 0.14g, meanwhile peak accelerations (PA) ranged between 0.002g and 0.31g. For all analyzed earthquakes, Fig.7 shows epicenters location and histograms for Magnitude, peak ground acceleration and peak acceleration.



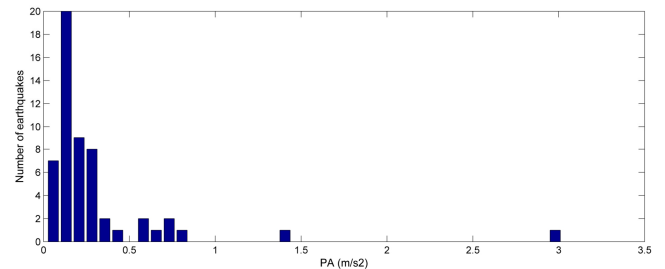
(a) Epicenters



(b) Magnitude histogram



(c) PGA histogram



(d) PA histogram

Fig.7 Properties of analyzed seismic events

## 7 The 2010 Chile Earthquake

From all the seismic events studied, only the  $M_w=8.8$ , 2010 Chile Earthquake caused visible damage on the building and permanent variations of its modal parameters. Despite this and the strong shaking of the building during the event (Maximum ground acceleration of 0.14g and structural acceleration of 0.31g), the structure only suffered minor and moderate damage, which was concentrated in non-structural elements (partition walls, ceilings, etc.), in addition to some visible shear cracks on the perimeter façade elements, Fig.8.



Fig.8 Observed damage after the 2010 Chile Earthquake

## 8 Results of the MIMO Analysis

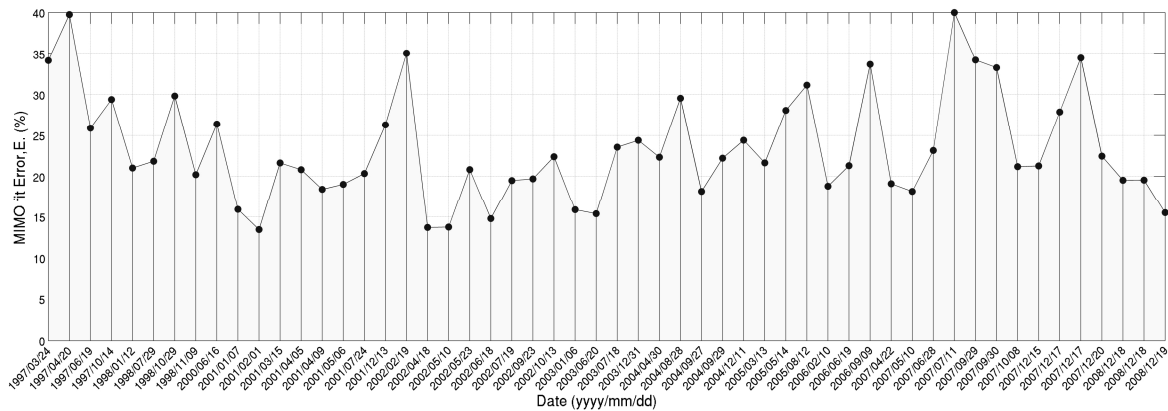
Before presenting the results of MIMO identification analysis on the seismic records, Table 2 shows the average modal parameters of the building, determined from ambient vibration records and presented on previous studies of the building ([3], [4], [5] and [6]).

**Table 2** Average modal parameters for ambient conditions. (Source: [5])

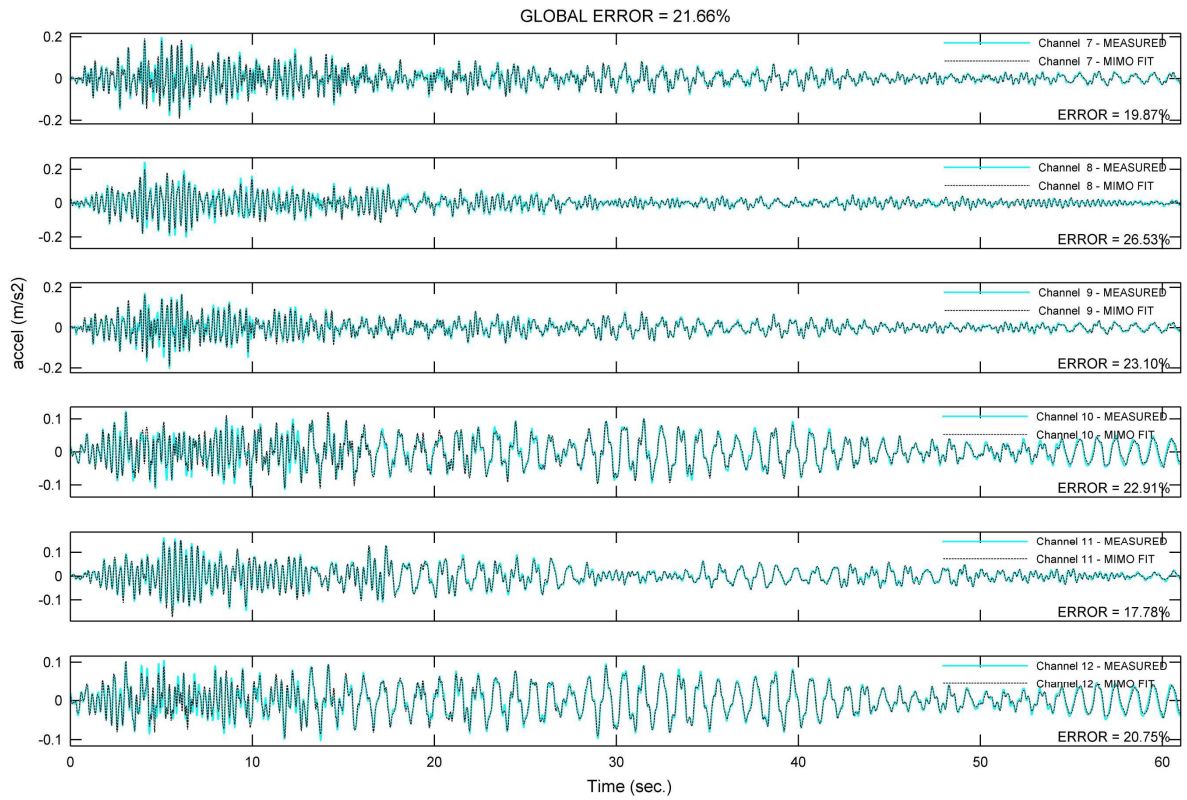
Mode	Frequency [Hz]	Damping Ratio ( $\xi$ ) [%]	Modal shape orientation
1	1.04	1.1	Translational. East-West.
2	1.07	1.0	Translational. North-South.
3	1.63	0.6	Torsional.
4	3.60	1.5	Translational. East-West.
5	3.57	1.5	Translational. North-South.
6	4.8	1.2	Torsional.

For the processing of the seismic records with the MIMO Identification algorithm, some important considerations were taken into account: The input of the identification algorithm was considered as the records of the four sensors located at the base of the structure, including those oriented vertically. As for the output of the MIMO algorithm, only the records corresponding to sensors 7 to 12 were taken into account, since the presence of channels 5 and 6 (corresponding to sensors located at ground level) in the process usually led to less accurate results for the damping and frequency determination. Finally, the MIMO identification algorithm analyzed only the strong motion stage of each seismic event, defined for this purpose as the time between the arrivals of the S-wave until a few seconds (approximately 5 to 10 times the main period of the building) after base accelerations have returned to normal ambient amplitudes

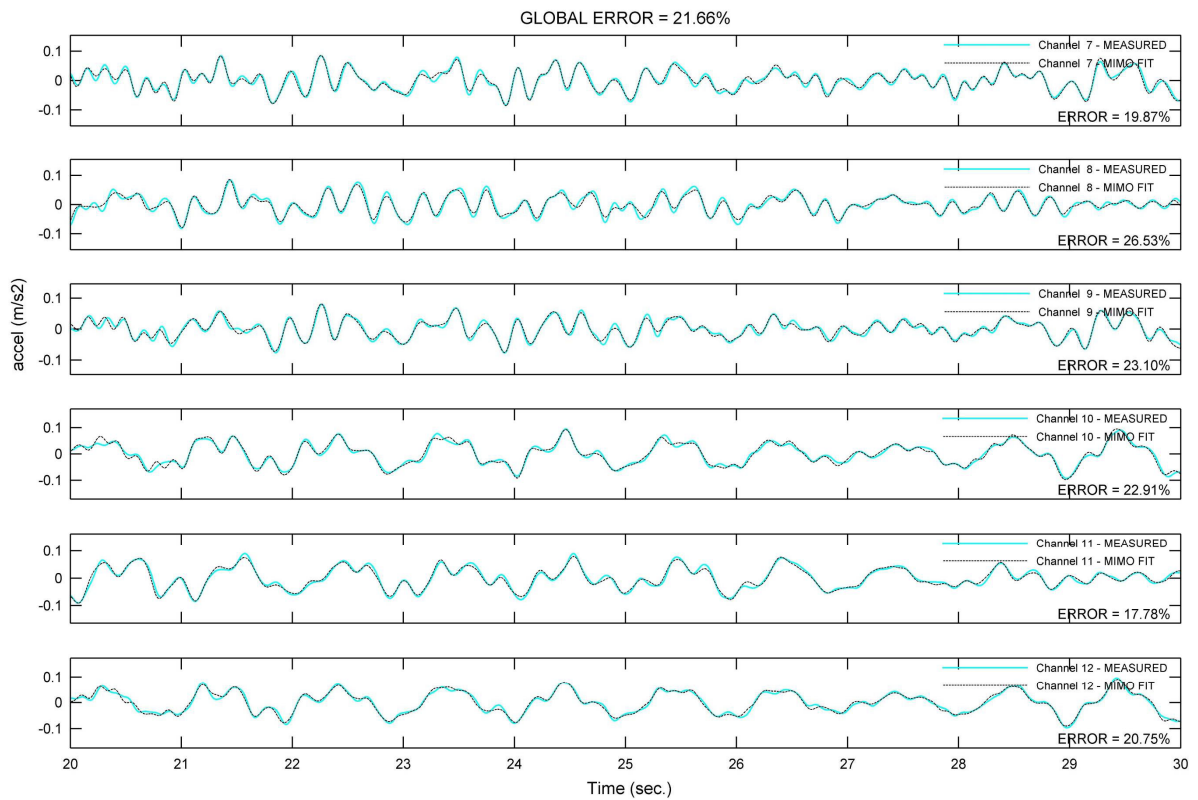
From the initial processing of each seismic record, identification analyses led to MIMO goodness of fit error ( $E$ , Equation (3)) ranging between 13% and 40%, which fall within acceptable ranges according to previous studies on the MIMO identification techniques that use accelerations as a reference response parameter.



**Fig.9** MIMO goodness of fit error for each seismic record



**Fig.10** Seismic event 2001/03/15. Recorded response vs. corresponding estimation from MIMO identification process



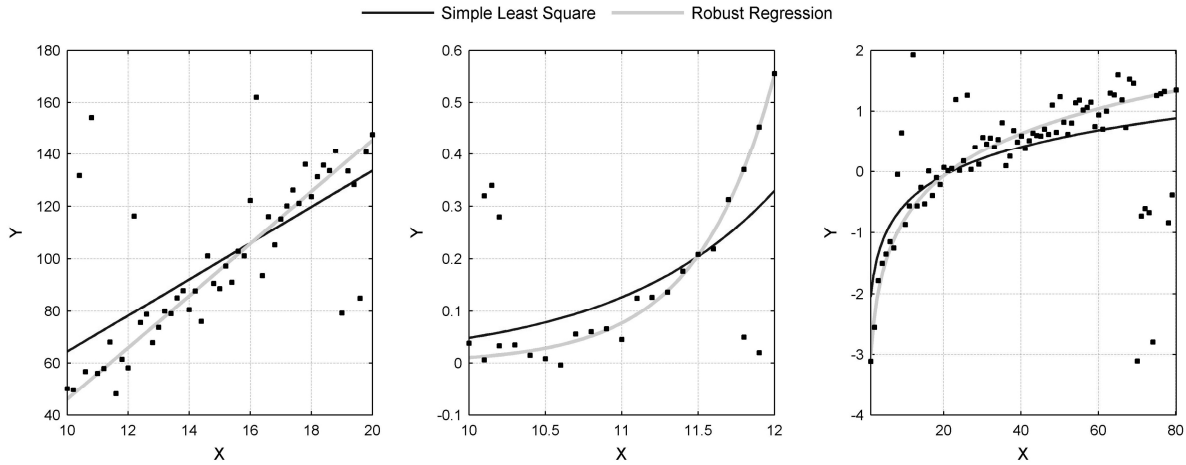
**Fig.11** Seismic event 2001/03/15. Recorded response vs. corresponding estimation from MIMO identification process. Zoom

An important part of this study is the analysis of the correlation between identified modal parameters and the corresponding severity of shaking of the building. Since the latter doesn't have an unique definition, this had to be specified during the study. A number of criteria were tested to evaluate the severity of shaking, being the definition of "modal amplitude" the one that best correlated to the identified modal parameters (modal frequency and damping ratio). This criterion involves filtering the seismic records around the natural frequency that is being analyzed, and then calculating its root means square value.

$$RMS_{\text{modal},j} = \sqrt{\sum_t \left( \left[ \tilde{a}_{19_{\text{th floor}}}(t) \cdot \tilde{\theta}_j \right]_{[f_j - \Delta f_j, f_j + \Delta f_j]} \right)^2} \quad (4)$$

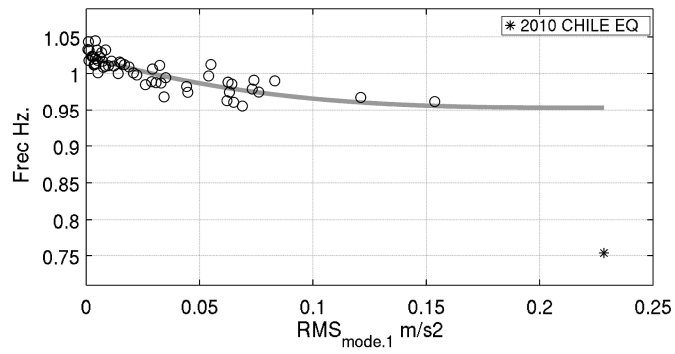
For all analyses, only modal parameters associated with the first 4 translational modes of the building (modes 1, 2, 4 and 5) were considered, since they are the ones with the highest participation on the structural response during seismic events. The torsional modes (3 and 6), had a small participation in all the analyses.

Fig.13 and Fig.14 show correlation graphs between the severity of shaking and the average modal parameters identified for each seismic record. A tendency line is included on each graph, resulting from a robust regression analysis of the data ([8]). As a general description, the robust regression analysis used in this study corresponds to a weighted least-square technique, where weight coefficients are assigned to each data point to minimize the influence of outliers. A comparative example between the results of a simple least-square analysis and a robust regression analysis is presented in Fig.12.

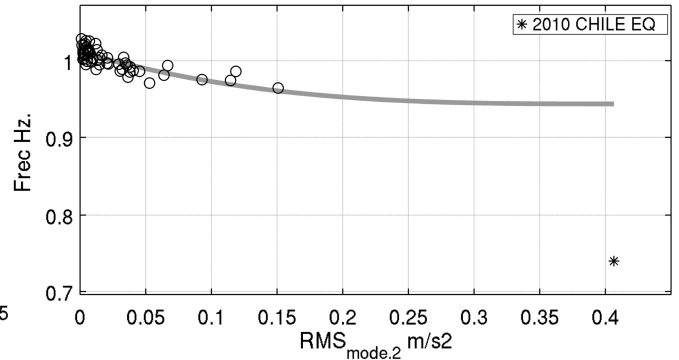


**Fig.12** Comparative example. Simple Least Square Regression vs. Robust Regression.

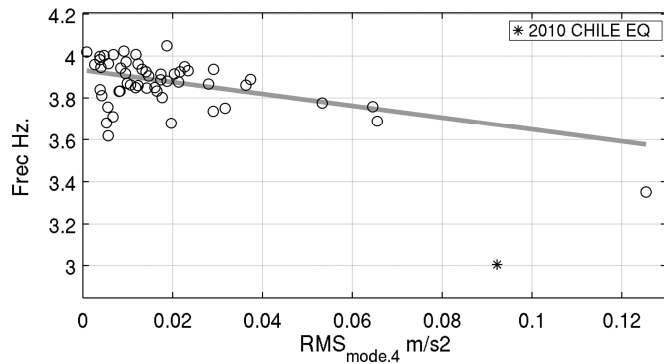




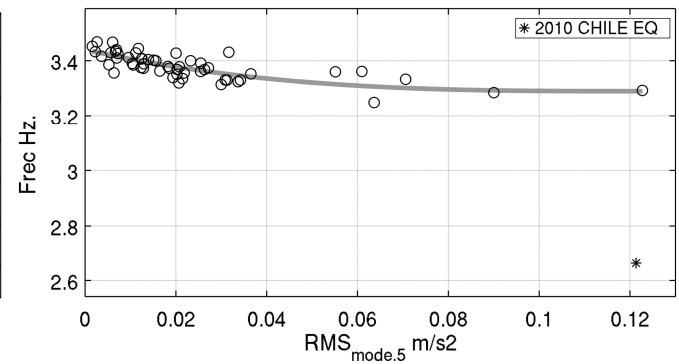
(a) Mode 1



(b) Mode 2

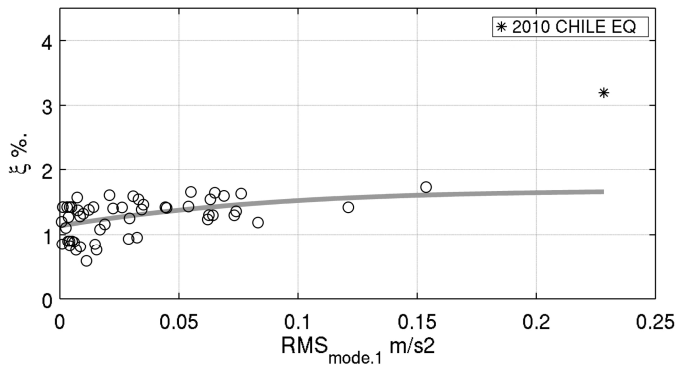


(c) Mode 4

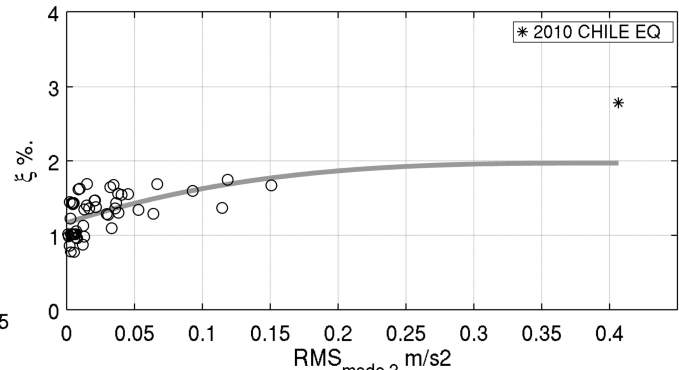


(d) Mode 5

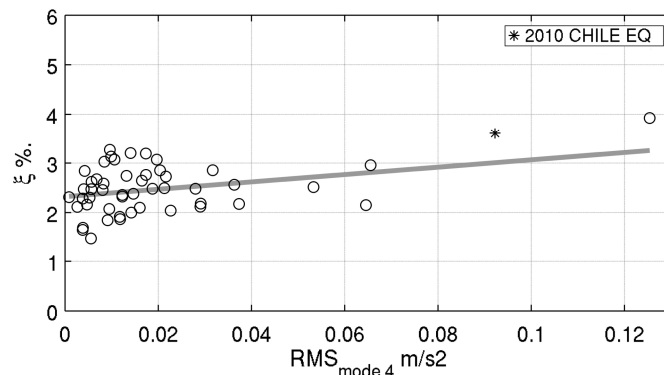
**Fig.13** Identified modal frequencies vs. corresponding modal amplitude



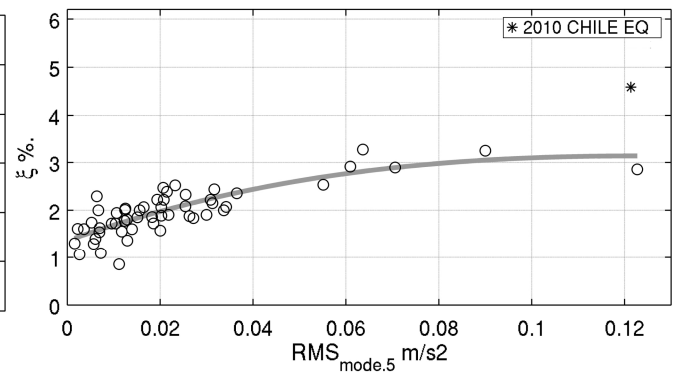
(a) Mode 1



(b) Mode 2



(c) Mode 4



(d) Mode 5

**Fig.14** Identified modal damping ratios vs. corresponding modal amplitude



In Fig.13 and Fig.14, results for the Mw=8.8 Chile Earthquake are especially highlighted on each graph. The regression analyses considered these results as outliers so they are not considered in the determination of the tendency lines.

From the analysis of Fig.13 and Fig.14 the correlation between the “modal amplitude” of the building and the identified modal parameters becomes clear. On one hand, modal frequencies show decreasing values when the severity of shaking increases, reaching values up to 4% and 9% for low to medium intensity earthquakes and, on the other hand, modal damping ratios display the opposite behavior, with corresponding tendency lines showing maximum relative increases between 40% and 120%.

During the 2010 earthquake, the biggest temporary variation of the modal frequencies took place, decreasing up to 35% on average for translational modes. Table 3 shows modal frequencies at three different times during the seismic event, these results were obtained by dividing the seismic record into small windows and identifying the modal parameters, using the MIMO algorithm, for each one of them.

**Table 3** Identified modal frequencies during 2010 Chile Earthquake

Mode	Start [Hz]	Minimum [Hz]	End [Hz]	Relative difference <sup>3</sup> [%]	
				Minimum	End
1	0.99	0.64	0.79	35.2	20.1
2	0.98	0.61	0.74	37.8	24.8
4	3.72	2.44	3.02	34.6	19.0
5	3.36	2.31	2.65	31.1	21.2
Average				34.7	21.3

Furthermore, the modal frequencies of the building changed permanently after this event, decreasing on average an 18% for the first modes. This permanent change was verified using ambient vibration right after the earthquake event. Table 4 presents identified modal frequencies and damping ratios (obtained through Stochastic Subspace identification technique over ambient vibration records).

**Table 4** Evolution of modal properties for ambient conditions. Before and after the earthquake

Mode	BEFORE EQ		AFTER EQ	
	Freq [Hz]	Damp (ξ) [%]	Freq [Hz]	Damp (ξ) [%]
1	1.01	0.6	0.84	0.6
2	1.03	0.7	0.86	0.6
3	1.54	0.6	1.22	0.8
4	3.45	1.1	2.91	1.1
5	3.44	1.2	2.86	1.1
7	4.62	1.1	3.67	1.1

For modal damping ratios, no distinguishable change can be appreciated between the two analyzed stages, because the observed differences fall within expected error for the employed identification technique (SSI, [12]).

## 9 Conclusions

In this study, a 22 story reinforced concrete shear wall building was analyzed to determine the correlation between its modal parameters and the severity of shaking during seismic events. The identification of the modal parameters was performed by a parametric Multiple Input- Multiple Output algorithm, which was developed and validated during this study.

From the testing of different criteria, a “modal amplitude” definition was chosen to quantify the severity of shaking, because it showed the best correlation to the identified modal parameters. For modal frequencies, decreasing values with the “modal amplitude” were observed, with corresponding tendency lines showing a maximum decrease between 4% and 9% for small to medium intensity earthquakes. As for the modal damping ratios, their values tend to increase with the severity of shaking, reaching a maximum relative increase, according to their tendency lines, between 40% and 120%

<sup>3</sup> Relative to corresponding identified modal frequency at the start of the seismic event. For minimum:

$$\text{Rel\_diff}_{\text{mode } j, \text{ min}} = \frac{(f_{j, \text{ start}} - f_{j, \text{ min}})}{f_{j, \text{ start}}} \cdot 100$$

From all the seismic events analyzed during this study, the Mw=8.8, 2010 Chile earthquake was the only one that caused visible damage of the structure, and a permanent change of modal parameters. During the seismic event, modal frequencies decreased up to 35% from initial values. Shortly after the event, identified modal frequencies showed an 18% permanent decrease.

Results of this research paper are representative indicators of the behavior of modal parameters during and after seismic events for a wide range of intensities.

### **Acknowledgement**

The Civil Engineering Department of the University of Chile and the Chilean Council for Research and Technology, CONICYT Fondecyt Project # 1070319 supported this research paper. The authors would also like to thank the Chilean Chamber of Construction and Civil Engineer Pedro Soto for their collaboration and support in the development of this study.

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