

Background Report

Influence of Geometric Nonlinearities on the Seismic Response and Design of Bridge Structures

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ABSTRACT

Geometric nonlinearities have been seen to have an important influence on the inelastic response of many structures subjected to earthquake motions. Design procedures to account for these so-called **P-Δ** effects are relatively simple, relying in the case of building structures on a combination of drift limits, proportioning requirements and strength enhancements. Currently, the *Bridge Design Specification Manual* (Caltrans, 1990) does not require consideration of **P-Δ** effects. None the less, Caltrans has adopted special design requirements for several recent projects in which geometric nonlinearities are explicitly considered. Little information is available regarding the reliability of such design criteria in general and the special circumstances that may occur for bridge structures in particular. This background report is intended to summarize some of the bases and limitations of methods for treating **P-Δ** effects in seismic design to provide insight into the inelastic seismic behavior of simple bridge structures acting under the influence of geometric nonlinearities, and to assess in a preliminary sense design criteria, including recently proposed Caltrans procedures, that can be used to compensate for **P-Δ** effects.

The first three sections of this report introduce various definitions, concepts and concerns related to geometric nonlinearities that may be of particular interest to readers who are not familiar with the literature on this topic. Following a brief introduction, the second section summarizes recent analytical literature related to the effect of geometric nonlinearities on the inelastic seismic response of structures. The third section reviews literature related to design considering geometric nonlinearities and the various practices stipulated in design codes for considering **P-Δ** effects.

The fourth section presents the results of a new set of nonlinear dynamic analyses which suggest ranges of design variables where proposed Caltrans and other design procedures may be used conservatively, and others where they produce highly unconservative results. The effect of varying (a) structural properties such as period, lateral strength, and restoring force characteristics, (b) intensities of geometric nonlinearities, and (c) the ground motion intensity, frequency content and duration are considered explicitly in the preliminary analyses presented. A statistical evaluation of the numerical results suggests that reliable empirical relations can be developed for use in design. The fifth section suggests an alternative design approach based on the analytical results. The last section summarizes the present study and suggests areas for further research.

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INTRODUCTION

The Bridge Design Specifications Manual (Caltrans (1990)) does not require consideration of geometric nonlinearities. These so-called "**P- Δ** effects" have been neglected on the basis of several observations. First, earthquake damage to reinforced concrete bridges is generally attributed to non-ductile details, insufficient shear reinforcement in members and joints, and inadequate anchorages. For such brittle modes of failure, geometric nonlinearities contribute little. Second, computed design displacements are generally small. In such cases, **P- Δ** effects are assumed inconsequential as long as they do not jeopardize static stability. Thirdly, building code provisions in the U.S. have until recently also ignored **P- Δ** effects.

The ductile detailing requirements currently prescribed by Caltrans will avoid most premature modes of failure. However, flexible bridge structures or bridges constructed in severe seismic settings may now be able to develop displacements large enough for **P- Δ** effects to become important. In recognition of this, Caltrans has begun to account for **P- Δ** effects explicitly in the design of several structures in the San Francisco bay area.

For example, for the San Francisco area double deck viaducts computed lateral displacements must not exceed 1% of the height of the structure when static lateral loads are applied resulting in a base shear equal to 25% of the weight of the structure. For a dynamic response spectrum analysis, displacements computed setting the Z (ductility and importance) reduction factor to unity should not exceed one-sixth the dimension of the structure's columns in the direction under consideration. To estimate displacements realistically, cracked section properties are used in analysis. These special design provisions were based on recommendations of the Peer Review Panel established for the San Francisco area viaducts.

A more detailed and technically based procedure for considering **P- Δ** effects was developed by Caltrans for the design of the new Terminal Separator structure in San Francisco. For this structure, the design moments in the columns are amplified to account for **P- Δ** forces acting on the structure. Thus, the incremental design moment M_{Δ} , used for this structure to compensate for **P- Δ** effects is given by:

$$M_{\Delta} = [1 + \mu_{\Delta}]P\Delta_y/\theta \quad (1)$$

in which P is the axial load acting on a column;
 u_m is the maximum lateral displacement developed by the column;
 μ_Δ is the displacement ductility factor ($= u_m/\Delta_y$);
 Δ_y is the lateral displacement at first yield; and
 θ is a factor to reduce the severity of the $P-\Delta$ effect,

These moments are added to conventional design moments obtained ignoring geometric nonlinearities. For most values of θ , the lateral displacement at which the $P-\Delta$ effect is evaluated using Eq. 1 does not correspond to the full design ductility factor. With θ equal to 2, for example, Eq. 1 results in roughly the same energy being absorbed by elasto-plastic systems with and without $P-\Delta$ effects for static loading applied in one direction (for clarification see Fig. 1 and later discussions on this matter). At a maximum ductility of 4, the displacement at a ductility of only $(1 + 4)/2 = 2.5$ would be used for design to compute $P-\Delta$ effects.

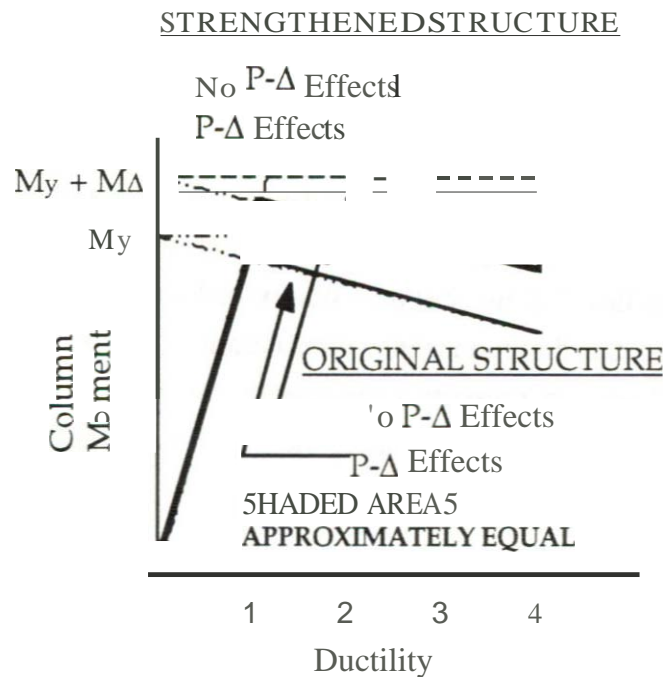


Fig. 1 Increase in Strength to Compensate for $P-\Delta$ Effects; Shown for $\theta = 2$ in Eq. 1. Resulting in Approximately Equal Energy Absorption for Structures with and without $P-\Delta$ effects (after Pauley, 1978)

The actual **P-Δ** moment, $M_{P-\Delta}$, at the maximum displacement of the system is given by:

$$M_{P-\Delta} = Pu_m = P\mu_{\Delta}\Delta_y \quad (2)$$

For Eqs. 1 and 2 to be equal, C must be $(\mu + 1)/\mu$; or for μ equal to 4, C must be 1.25. Because of the sensitivity of the inelastic dynamic response of structures to **P-Δ** effects, C in Eq. 1 was taken for the design of the Terminal Separator to be 1.5, rather than 2. For a maximum displacement ductility factor of 4, this corresponds to a design displacement for computing **P-Δ** effects equal to 3.33 times the yield displacement.

Unfortunately, little quantitative information is available regarding the reliability of such design criteria. This brief report summarizes some of the bases and limitations of Eq. 1, and provides some results of nonlinear dynamic analyses to help assess its viability for design purposes.

The next section of this report introduces various definitions and concepts related to **P-Δ** effects and summarizes recent analytical literature. The third section reviews literature related to design considering geometric nonlinearities and practices stipulated in design codes for considering **P-Δ** effects. The informed reader may wish to skip these two sections. The fourth section presents the results of a new set of nonlinear dynamic analyses which suggest ranges of design variables where Eq. 1 may be used conservatively, and others where it produces unconservative results. The fifth section suggests an alternative design approach based on the analytical results which may be used in cases where Eq. 1 is unconservative. The last section summarizes the present study and suggests areas for further research.

RESPONSE OF STRUCTURES INCLUDING P- Δ EFFECTS

Surprisingly little information is available in the literature related to the effects of geometric nonlinearities on inelastic seismic response. Many studies have been directed at refining the analytical or numerical formulations that can be used to account for geometric nonlinearities. For example, Eq. 2 is based on small deflection theory, and several studies have analytically investigated the importance of higher order terms. In other analytical studies, the effect of geometric nonlinearity on the distribution of internal forces along a member have been examined. For design purposes such refinements are generally not warranted, since large displacements near incipient member buckling are to be avoided.

Many studies of inelastic seismic response have incorporated P- Δ effects since the computer programs utilized automatically permit their consideration. However, few studies have attempted to understand the sensitivity of inelastic response to geometric nonlinearities, or to identify characteristics of the structure or ground motion that control this sensitivity. One of the main reasons for this situation is that the inelastic dynamic response of a structure with negative post-yield tangent stiffness, as may occur when P- Δ effects become dominant, is highly unstable. This recognition has led most researchers to recommend that structures be designed so that P- Δ effects do not become important; i.e., they possess sufficient stiffness to limit lateral displacements.

When P- Δ effects are to be considered a simple geometric model is usually employed in the analysis. Such a model is shown in Fig. 2.

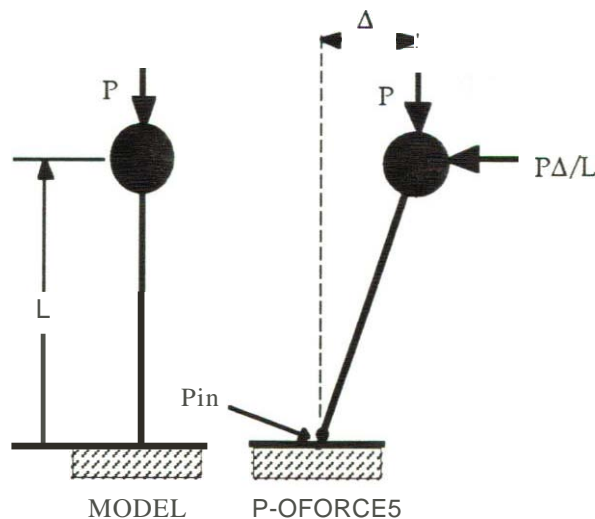


Fig. 2 Simplified Model of P- Δ Effects

In the absence of any structural resistance, a lateral load equal to $P\Delta/L$ must be applied to oppose the displacement of the structure. This force is linear with respect to lateral displacement. However, the lateral stiffness associated with this system is negative, since the restoring force must be applied in a direction opposing the observed displacement. This geometric stiffness can be employed for any single degree of freedom system, regardless of its mechanical characteristics.

For simplicity, the mechanical characteristics of a ductile structure are often assumed, in the absence of geometric nonlinearities, to be elasto-perfectly plastic (see Fig. 3). Once such a structure displaces laterally in the presence of axial loads, $P-\Delta$ induced forces (Fig. 2) will reduce the strength of the structure remaining to resist externally applied loads. Geometric nonlinearities will also reduce the apparent lateral stiffness of the structure in the elastic range, and may result in a negative post-yield tangent stiffness, as shown in Fig. 3. It should be noted that the yield displacement of the structure does not change because of the geometric nonlinearities.

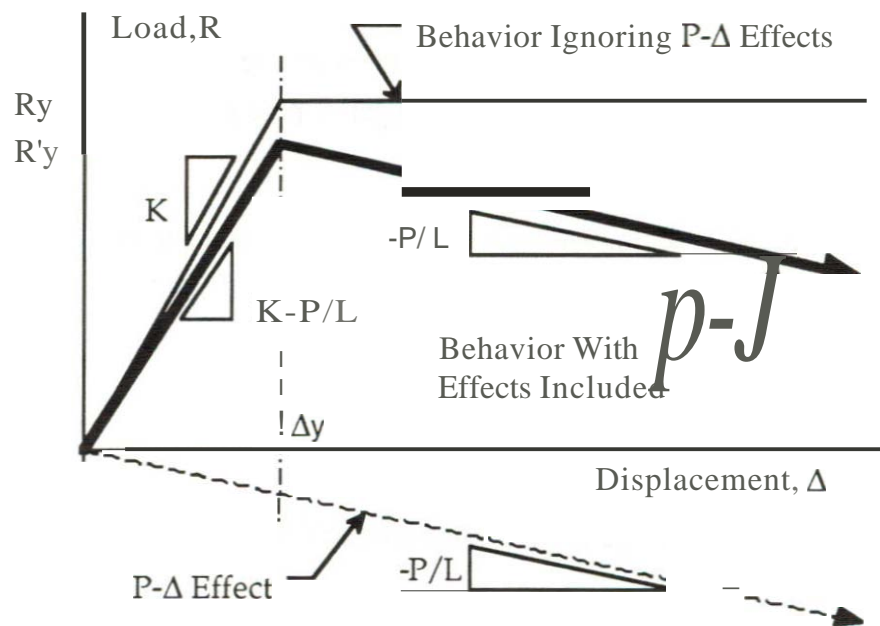


Fig. 3 Schematic Effect of Geometric Nonlinearities on Mechanical Characteristics of Structure

The actual post-yield tangent stiffness (and strength) of a structure will depend on its mechanical characteristics in the absence of geometric nonlinearities. Highly redundant systems, like building frames, often exhibit considerable deformation hardening (positive post-yield tangent stiffnesses) as a result of the redistribution of forces that occurs during

the progressive development of plastic hinges needed to form a collapse mechanism. The literature related to building frames often cites ultimate strengths 300 to 500% greater than the specified design yield strength. Such high amounts of deformation hardening are not likely to develop in simple bridge structures (i.e., the ratio of plastic to ultimate (nominal) moments considered for design ranges from 1.3 to 1.5). In any event, the mechanical characteristics of most structures can be represented for discussion purposes by an "effective" elasto-perfectly plastic curve.

The idealization of geometric nonlinearities must become slightly more complex for multilevel structures. Limiting the current discussion to simple single level bridge structures, the model described will suffice.

Several studies (e.g., Newmark (1971), Mahin and Bertero (1981), Bertero (1976), MacRae (1991)) have investigated the influence of the rate of deformation hardening on inelastic response. Generally, it has been found that small positive post-yield tangent stiffnesses reduce maximum displacements slightly and residual displacements substantially. On the other hand, even very small negative values for the post-yield tangent stiffness results in large increases in both maximum and residual displacements. Four intuitive reasons for this adverse behavior when negative post-yield tangent stiffness occurs can be advanced.

First, considering the incremental equations of motion for an *undamped* single degree of freedom system at time increment 'i':

$$m\Delta a_i + \Delta R_i = -m\Delta a_{g_i} \quad (3)$$

in which

m is the mass of the system;

ΔR_i is the change in nonlinear restoring force during the time step in question;

Δa_i is the change in relative acceleration of the mass; and

Δa_{g_i} is the change in the acceleration of the ground.

Recognizing that the total acceleration Δa_{t_i} of the mass is the sum of Δa_i and Δa_{g_i} , Eq. 3 can be rewritten as:

$$m\Delta a_{t_i} + \Delta R_i = 0 \quad (4)$$

or

$$\Delta a_{tj} = -\Delta R_j/m \quad (5)$$

Thus, for a simple yielding system that is elasto-perfectly plastic, the term ΔR_j is zero and the change in acceleration of the system must also be zero. As a result, the system would maintain the same acceleration and displacements would increase quadratically. For a system with a positive post-yield tangent stiffness, the term ΔR_j is positive resulting, according to Eq. 5, in reductions in acceleration during each step the structure yields. Since the accelerations reduce, the displacements would be expected to reduce accordingly.

For a system with a negative post-yield tangent stiffness, the term ΔR_j is negative and the structure's acceleration would increase during each step. Since the accelerations increase, the displacements would increase as well. Thus, a descending post-yield stiffness will inherently tend to increase the displacements of a structure. Because of the quadratic relation between accelerations and displacements, these increases in acceleration are expected to have a disproportionate effect on displacements.

The second reason for increased response of a system with negative post-yield tangent stiffness is that the structure's strength is no longer the same in each direction of motion. Modifying Fig. 3 for the case of cyclic elasto-perfectly plastic mechanical behavior, one obtains the plot shown in Fig. 4. For example, consider the loading path that takes the structure to point A in Fig. 4. If the structure unloads at this point, it must attain the force corresponding to point C for yielding to commence in the opposite direction. It should be noticed that the yielding resistance of the structure for loads toward the origin has been increased by the $P-\Delta$ effects. If, on the other hand, loading continues from point A toward point B, the effective yielding resistance has been reduced by the $P-\Delta$ effect. Thus, the structure is weaker for motion away from the origin, and stronger for motions that would return it toward its initial position. The same characteristics occur if the structure initially move to the left in Fig. 4, rather than to the right. Once yielding occurs in one direction, subsequent yielding of the structure will tend to be in that same direction because of the geometric nonlinearities.

Newmark (1971), among others, has studied the ductility demands on a structure that is weaker in one direction than in the other. He observed that demands will tend to be greater for these unsymmetrical systems than for otherwise similar symmetric structures having the weaker strength in both directions. This is true even for systems without

geometric nonlinearities, i.e., systems with no deformation softening. Intuitively, this increase in ductility demand may be explained by the observation that when the structure deforms elastically toward the stronger side it can store far greater elastic strain energy than if it yielded at a lower load. Thus, it can rebound toward the weaker side with far greater energy, producing potentially greater ductility demands. As a result, once a structure subject to $P-\Delta$ effects displaces away from its origin, *incremental* ductility demands for individual acceleration pulses within an earthquake record will tend to be greater in the weaker direction (heading away from the origin), than they would be toward the stronger direction (returning the structure to the origin).

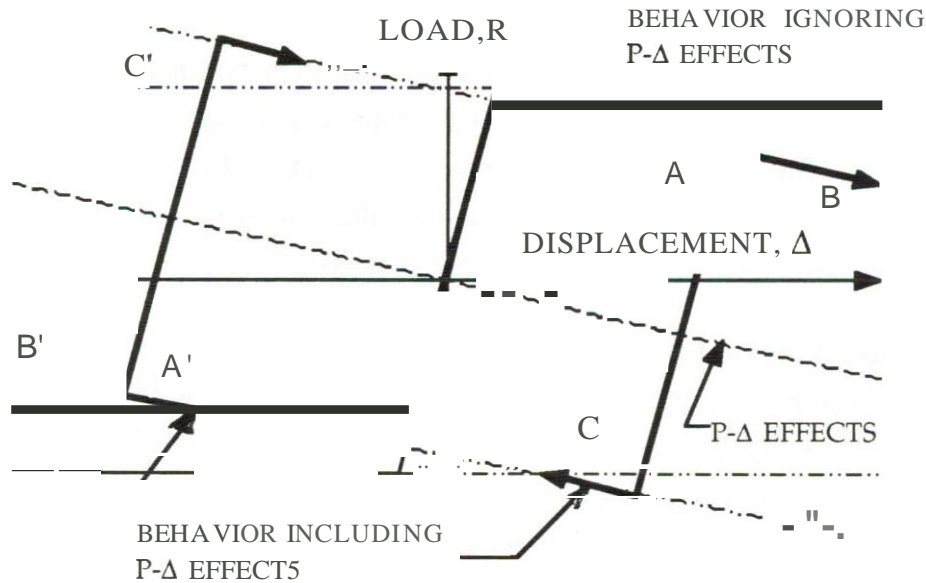


FIG.4 Effect of Geometric Nonlinearities under Cyclic Loading

A fourth reason one might advance to explain the potentially increased displacements for systems with geometric nonlinearities is that $P-\Delta$ effects increase the period of a structure (since the effective elastic stiffness is reduced). Elastic spectral displacements would then be expected to increase resulting in larger displacements for structures subject to $P-\Delta$ effects.

In much of the design oriented literature one finds mention of the displacements that may occur during inelastic response. For long period structures (relative to the predominant period of the ground supporting the structure) it has been established (e.g., Newmark (197\)) and Mahin (1981)) that a good estimate of maximum displacements can be made from the elastic spectral displacements corresponding to the initial elastic period of the

structure and a value of damping corresponding to the elastic range of response. Newmark showed that the shape of the hysteretic loops has only a marginal effect on the maximum inelastic displacements so long as the envelop curves are symmetric and stable (no reduction of strength occurs at points of maximum displacement as a result of cycling). For example, it has been well established by Newmark (1971), Mahin (1981) and others that the average maximum displacement experienced by stiffness degrading systems (representing reinforced concrete structures) is virtually indistinguishable from those of bilinear hysteretic systems (representing steel structures).

This observation has led many to assume that $P-\Delta$ effects have little influence on the overall energy dissipation capacity of structures with geometric nonlinearities. Thus, Pauley (1978) suggests that the energy dissipated during displacement cycles with equal amplitudes in each direction is unaffected by geometric nonlinearities. This is shown conceptually in Fig. 5 where the shaded area for the system including $P-\Delta$ effects equals that of the system without geometric nonlinearities. This equality of energy dissipation capabilities has led some to assume that the maximum displacements of structures with and without $P-\Delta$ effects should be about the same.

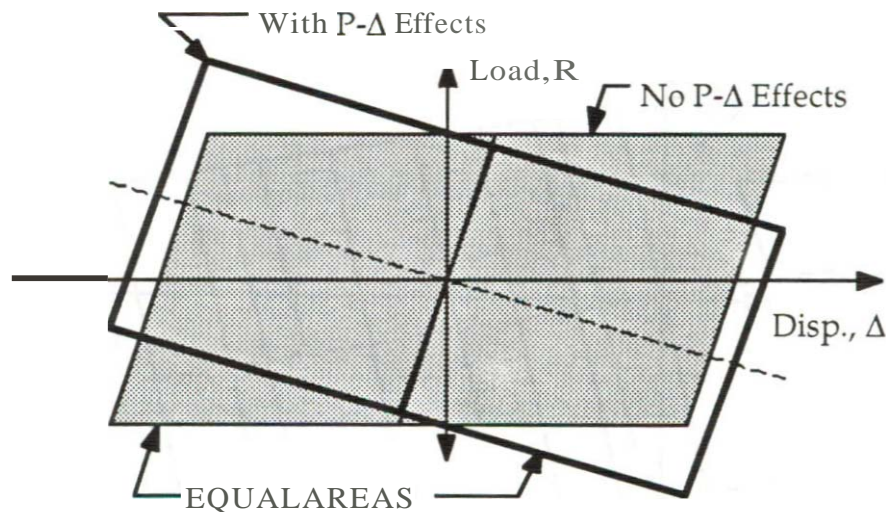


Fig. 5 Equal Energy Dissipation for Cycles with Equal Displacement Amplitudes

However, as indicated above one would not expect the maximum displacements to be the same in both directions of motion. Newmark (1971) also stipulates that for his observations to be valid the post-yield tangent stiffness must not be negative (and that ductility demands and damping values should not be large). Similarly, Wakabayashi

(1986) indicates that equal energy or displacement assumptions cannot be used conservatively to predict maximum inelastic displacements from elastically computed displacements when the post-yield tangent stiffness is negative, as happens when geometric nonlinearities dominate.

The typical displacement history for systems with significant $P-\Delta$ effects is biased in one direction. Clearly, structural displacement histories will be sensitive to the sequence and intensity of individual acceleration pulses in the earthquake record. However, typical hysteretic loops show ever increasing amounts of permanent offset during the response, leading to what is called an "incremental," "crawling" or "ratcheting" type failure mode (Bertero (1976)). This offsetting behavior is illustrated in Fig. 6. Displacements in such cases are substantially larger than predicted elastically (or inelastically using elasto-perfectly plastic hysteretic characteristics). This is true even where the tangent stiffness is small. For example, Mahin (1976) found that displacements can double or triple (or even result in static collapse of the structure) for negative slopes as small as 1% of the initial elastic stiffness. This was especially true for systems with ductility demands (ignoring $P-\Delta$ effects) greater than 4 to 6. For smaller ductility demands, smaller increases would be expected due to the less severity and fewer number of yield excursions.

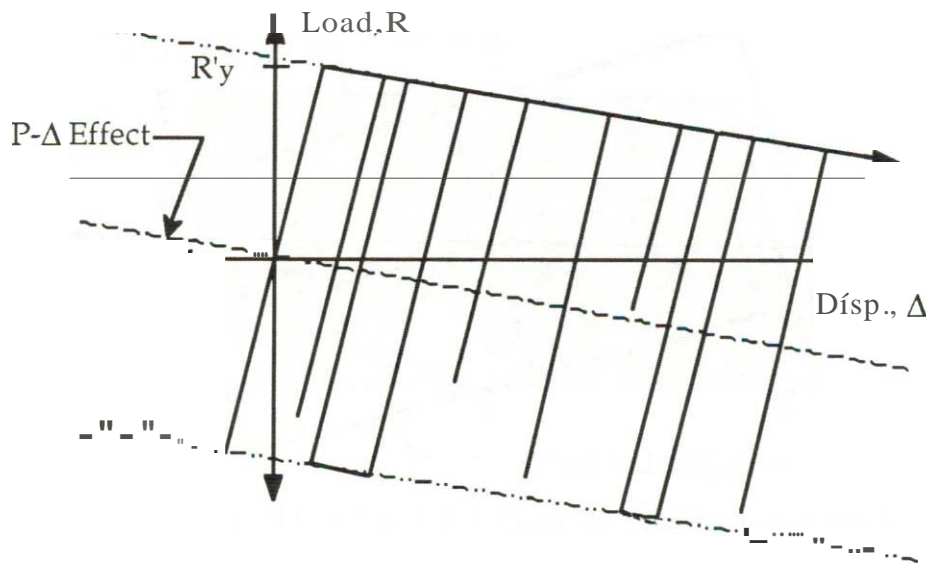


Fig. 6 Bias in Displacement History Leading to Incremental Collapse Mode

It has been noted by MacRae (1991), Ziegler (1980), Mahin and Bertero (1981) and others that the residual deformations of inelastic structures diminish rapidly with even small positive post-yield tangent stiffnesses, but increase dramatically for negative values. For

elasto-perfectly plastic systems the residual displacement ranges roughly from 40 to 60 % of the maximum inelastic displacement of the structure. (Stiffness degrading systems perform better with residual displacements typically less than 30 % of the maximum displacement achieved.) For systems with small **P-Δ** effects, residual ductilities range from 90 to 100% of the maximum ductility developed (MacRae (1991)). This would have an important impact on operability and repairability following a design earthquake. Moreover, the **P-Δ** induced forces associated with the permanent displacements may reduce the strength of the structure to the point where it cannot satisfactorily resist aftershocks or other future earthquakes.

Because of the directional bias observed in systems subjected to **P-Δ** effects, one would expect that a main parameter influencing maximum displacement is the duration of shaking. This has been noted by many investigators [e.g., Bertero (1976), Jennings (1968), Mahin (1976), Newmark (1975), Sun et al. (1973), Takizawa (1980), Wakabayashi (1986)]. These studies show that **all** elasto-perfectly plastic systems subjected to **P-Δ** effects are inherently unstable and that, given a sufficiently long earthquake, will collapse.

Husid (1967 and 1968) and Jennings (1968) considered systems with elasto-perfectly plastic hysteretic restoring force characteristics, 2% viscous damping, and initial periods ranging between 0.5 and 2.0 secs. A variety of real and artificial ground motions was considered. Based on these analyses it was found that the expected time to collapse, $E(t_c)$, could be approximated by an expression of the form:

$$E(t_c) = e L [R_y / (BP)]^2 \quad (6)$$

in which B is a measure of earthquake intensity related to root mean squared acceleration divided by g ;
 e is an empirical coefficient;
 P is the weight of the system supported on the column;
 L is the height of the structure; and
 R_y is the initial yield capacity of the structure ignoring **P-Δ** effects,

Newmark (1971) discusses the reasons for the form of this expression. He stipulates that such expressions are valid only when the applied axial loads in the structure are far less than the structure's static lateral buckling load. It can be noticed from Eq. 6 that the expected time to collapse is proportional to R_y/BP . Here, R_y/P represents the base shear

coefficient for the structure at the onset of yielding. As such, $P-\Delta$ effects would be expected to be less severe for stronger systems (relative to the strength of the earthquake). These would have lower Z values (lower design ductility demands). The time to collapse is also inversely proportional to PIL , the slope of the post-yield tangent stiffness. Consequently, the smaller the $P-\Delta$ effect (PIL), the longer it will take the structure to collapse. For design one would thus try to minimize PIL .

Mahin (1976) examined the effect of aftershocks on the inelastic deformation demands of systems subject to geometric nonlinearities. This study showed that ductility demands more than doubled during the aftershock sequences considered.

Significantly, the time to collapse in Eq. 6 is not a function of period. Newmark (1971) showed that for periods near and below the limits considered in Housner and Jennings' study (0.5 sec.) inelastic displacements exceed elastic ones. One would assume that $P-\Delta$ effects would be more severe for these shorter period structures as a result. Ziegler and Mahin (1980) examined a broader range of elastic periods for elasto-perfectly plastic systems and, like Jennings (1968), found that ductility demands increased for large values of PIL , large durations of shaking and lower strengths (relative to the intensity of the ground motion). By considering shorter period structures, they also found that ductility demands increased with decreasing period.

Ziegler (1980) also studied the effects of different hysteretic models on time to collapse. In these studies simple stiffness degrading models were used, representative of reinforced concrete. This study suggested that $P-\Delta$ effects were far less important for stiffness degrading systems than for elasto-perfectly plastic ones. However, these studies were limited to cases where the reduction of apparent structural strength, because of $P-\Delta$ effects, would be less than 8% of the initial yield capacity at a ductility of 4. In addition, the hysteretic model utilized in these studies did not properly capture the effect of geometric nonlinearities on the elastic range of response at large displacements.

The reason stiffness degrading systems are believed to be less sensitive to $P-\Delta$ effects is that motion to return the structure to the origin is not resisted by the full elastic stiffness and amplified strength of the structure, but by a greatly reduced stiffness (see Fig. 7). Thus, it is far less difficult for the system to return to the origin along path A-B in Fig. 7, than if the system were elasto-plastic. The validity of these observations for more severe geometric nonlinearities remains to be assessed. Under these conditions, the system

will still have a significant elastic range (stiffness and pseudo-strength) in the reversed direction (near point A) due to the 'tilting' of the loops.

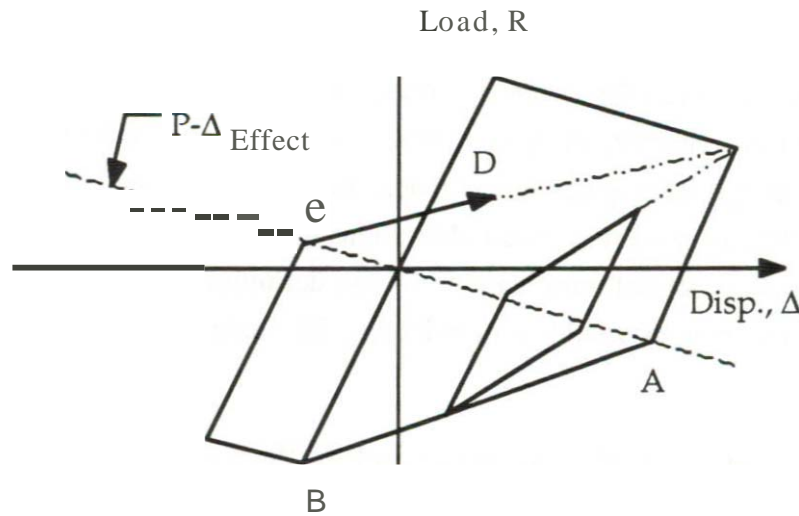


Fig. 7 Stiffness Degrading System with $P-\Delta$ Effect Included

Takizawa and Jennings (1980) also investigated the collapse characteristics of reinforced concrete frames (represented by a single degree of freedom system with stiffness degrading hysteretic characteristics). The focus of their study was to assess the effect of ground motion characteristics on collapse potential. They showed that peak acceleration was not sufficient to characterize the severity of response and that duration of shaking had a very significant influence for these systems. In particular, they found that short duration motions, in spite of very high acceleration intensities, exhibit little damage potential. Another important finding from this study was that the period of the structure had very limited influence on the ultimate inelastic response of the structures, but that yield strength and the severity of the geometric nonlinearity (magnitude of $-PIL$) were dominant factors. They also noted that stiffer structures have a larger margin of safety against collapse than do more flexible ones.

Some additional insight into the effect of the shape of the hysteresis loop on ductility demands was provided recently by MacRae et al. (1991). They show that some types of systems will collapse through excessive lateral displacements, if the duration of the ground motion is long enough. These systems are termed "unstable." Examples of this would be the bilinear systems considered herein with negative post-yield tangent stiffnesses. Some systems are "unconditionally stable," in that displacements are bounded

regardless of the duration of shaking. Examples of this are bilinear hysteretic systems with positive post-yield tangent stiffnesses. The definition of stability is based on an imaginary skeleton curve drawn halfway between the loading and unloading envelopes for a structural model. So long as this skeleton curve is positive for positive displacements (or negative for negative displacements) the system is termed stable. Negative slopes of the skeleton curve correspond to potentially unstable systems. The imaginary skeleton curve may be positive in some ranges of displacement and negative in others. Thus, some systems may be termed "conditionally stable." These definitions have not been fully evaluated at this time. However, an important ramification of these definitions is that classic model of a stiffness degrading system would be considered, for some loops, only conditionally stable.

It appears that a definitive investigation of **P- Δ** effects has yet to be carried out. The studies that have been done indicate that **P- Δ** effects can significantly increase ductility demands, especially for long duration earthquakes, for systems having large values of P/Δ , design ductility (μ_Δ or Z) factors equal 10 or greater than 4, and for structures with periods near or less than the predominant period of the ground. Thus far, analytical investigations have not developed a rational basis for design considering **P- Δ** effects.

DESIGN OF STRUCTURES TO RESIST $P-\Delta$ EFFECTS

A number of inelastic dynamic analyses have been carried out to assess $P-\Delta$ effects on actual structures and to devise design procedures and criteria. Nearly all of these have focused on building structures. In this section, some of these studies will be reviewed. In addition, the provisions of various building codes related to $P-\Delta$ effects will be presented and compared with evolving Caltrans criteria.

Results of Analytical Parameter Studies. -- In most analytical studies, geometric nonlinearities had little effect on well designed buildings. Typical of this is an early study by Powell and Rowe (1976) in which a number of ten story frames were analyzed for five different ground motions. They found only small increases in inelastic response when $P-\Delta$ effects were included in the analyses. The maximum inelastic inter-story drift computed for these frames was, however, only 1.2%.

Moss and Carr (1980) carried out a parameter study on 6, 12 and 18 story reinforced concrete framed structures. The results of their inelastic dynamic analyses indicated that, if the inter-story drifts obtained ignoring $P-\Delta$ effects were less than 1.5% of the story heights, then the maximum displacements including $P-\Delta$ effects were equal or slightly less than the values predicted ignoring $P-\Delta$ effects.

The small reductions in displacement were attributed to the shift of the structure's period. Since the strength of the structures analyzed was selected on the basis of the period computed ignoring $P-\Delta$ effects and the design spectrum monotonically decreased in amplitude with increasing period, this period shift resulted in structures stronger than would be required had the period been computed considering geometric nonlinearities. In an independent study of this effect, Andrews (1977) found that while disregarding geometric nonlinearities resulted in stronger structures, the amount of increase did not fully compensate for the loss in energy absorption capacity of the structure when $P-\Delta$ effects were applied. Thus, the minor reductions in displacement at small drifts with $P-\Delta$ effects included may be a ramification of the particular ground motion used in the analysis.

Moss and Carr (1980) also found that as drift indices (computed ignoring $P-\Delta$ effects) exceeded 1.5%, displacements computed accounting for geometric nonlinearities rapidly exceeded the values computed when $P-\Delta$ effects were discounted. For example,

the ratios of displacements with **P-Δ** effects to those without were approximately 1.5 and 2.5 for drift indices of 2% and 3%, respectively.

Moss and Carr suggest (as does Pauley (1978)) that it is more practicable to control excessive displacements in frame structures with important **P-Δ** effects by increasing strength than by increasing stiffness. It should be noted (Newmark (1971)) that this is not the case when **P-Δ** effects are unimportant; increases in strength have little effect on overall displacement. Where **P-Δ** effects become important, increasing strength expands the elastic range of response, thereby reducing the range over which the post-yield tangent stiffness may be negative. Moreover, Andrews (1978) argues compellingly that it is more effective in design to control **P-Δ** effects by limiting drifts.

Jury (1978) also carried out a study of six, twelve and eighteen story reinforced concrete frames. This study focused on the amount of strengthening needed for columns to achieve a strong column-weak girder design. The frames were designed in conformance with the New Zealand Standard Code of Practice, i.e., drifts at a ductility of 2.5 were limited to 1% of story height. He found that **P-Δ** effects were important and that lateral drifts in excess of 2.3% were sometimes attained. For some of his frames, **P-Δ** induced forces were as much as 50% of the strength of the structure. These structures, like those analyzed in the previously mentioned studies, tended to form plastic hinges in the beams. As a result, complete collapse mechanisms were not formed and it is unlikely that the structures developed negative post-elastic tangent stiffness.

Several studies have also been made related to steel frame structures. These are similar in their findings to those based on reinforced concrete frames. For example, Montgomery (1979) found that **P-Δ** effects need be considered only when ductility demands exceed 2, or when the base shear coefficient for the structure is less than 0.10.

Trjondro, Moss and Carr (1988) found that **P-Δ** effects were insignificant in mid-rise steel frames when the lateral design drifts were limited to 1.5% of the story height. Furthermore, they found that New Zealand design procedures (see later in this section) provide an adequate means of controlling and compensating for **P-Δ** effects so long as the design drift index is less than 1.7 to 2%. Typically, steel framed buildings are lighter, but more flexible than concrete buildings. Thus, one might expect a smaller magnitude for **PIL** in such structures, but a larger lateral displacement. Drift control was again found to be the key for minimizing **P-Δ** effects.

MacRae (1990) also studied design criteria for steel frame buildings subjected to $P-\Delta$ effects. In this case he used an assumption that the inelastic response of the frame would be similar to that obtained applying the design earthquake spectrum elastically and accounting for the softening effect of geometric nonlinearities. Simplifications in computing the distribution of lateral forces were introduced. Based on this procedure, acceptable frame performance was observed.

The above frames were designed in accordance with common building design practices which stipulate strong column-weak girder designs. The structures were midrise frames, and higher mode effects were found to be important in determining the temporal distribution of plastic hinges over the height of the building. These two factors made it unlikely that a complete collapse mechanism would form. Kelly (1977) studied cases of reinforced concrete frames where column hinging was intentionally induced. He found that the structures showed "a tendency towards continually increasing deformations in one direction, indicating progressive failure as the $P-\Delta$ moments reached high proportions of the ultimate column moments." He concludes his study saying "that multistory reinforced concrete structures relying on column hinge mechanisms are unsatisfactory for earthquake resistance when designed to code loading, or even twice code loading, in spite of ductile detailing. There is a significant risk that large deformations would initiate incremental $[P-\Delta]$ collapse•" These observations support the requirements in most building codes that promote the formation of strong column-weak girder collapse mechanisms.

As indicated previously, it is not expected that multistory buildings form a complete collapse mechanism. Even under monotonically (statically) applied lateral loads, a complete collapse mechanism is not often formed until quite large lateral deflections. As a result of the force redistribution associated with this successive formation of plastic hinges, the plastic strength of structures is often much greater than their ultimate design force (an increase of 300 to 500% is often cited in U.S. literature for buildings). This produces a significant post-yield strain hardening. Geometric nonlinearities must overcome this deformation hardening before the effective post-yield tangent stiffness becomes negative. Figure 8(a) conceptually plots the restoring force characteristics of a structure where $P-\Delta$ effects amount to about 50 % of the initial ultimate design strength of the structure at a displacement ductility of 8. In this case, $P-\Delta$ effects are of little practical consequence. Thus, most building codes treat $P-\Delta$ effects leniently, through a combination of (a) stipulated strong column - weak girder and other proportioning requirements that thwart the

formation of a complete collapse mechanism and result in a structure generally much stronger than required: (b) drift limits and (c) strength increases where $P-\Delta$ induced forces exceed some threshold.

The adequacy of such provisions for simpler building systems that are not likely to have such high over-strength and may form complete collapse mechanisms is uncertain. Similarly, the applicability of such provisions to bridge structures is uncertain, because of the lower degree of redundancy and the lower ratio of plastic to ultimate capacities. For example, Fig. 8 (b) schematically illustrates the behavior of a system with 50% increase in strength due to deformation hardening, where 50% of the initial strength is needed to resist $P-\Delta$ effects at a displacement ductility of 8. Clearly, $P-\Delta$ effects will have an important influence on the dynamic response, and a negative post-yield tangent modulus develops. The simplified analyses described in the previous section of the report are more likely to be more applicable in such cases than the building analyses cited in this section; provided, of course, that "effective" strength and deformation softening parameters are utilized.

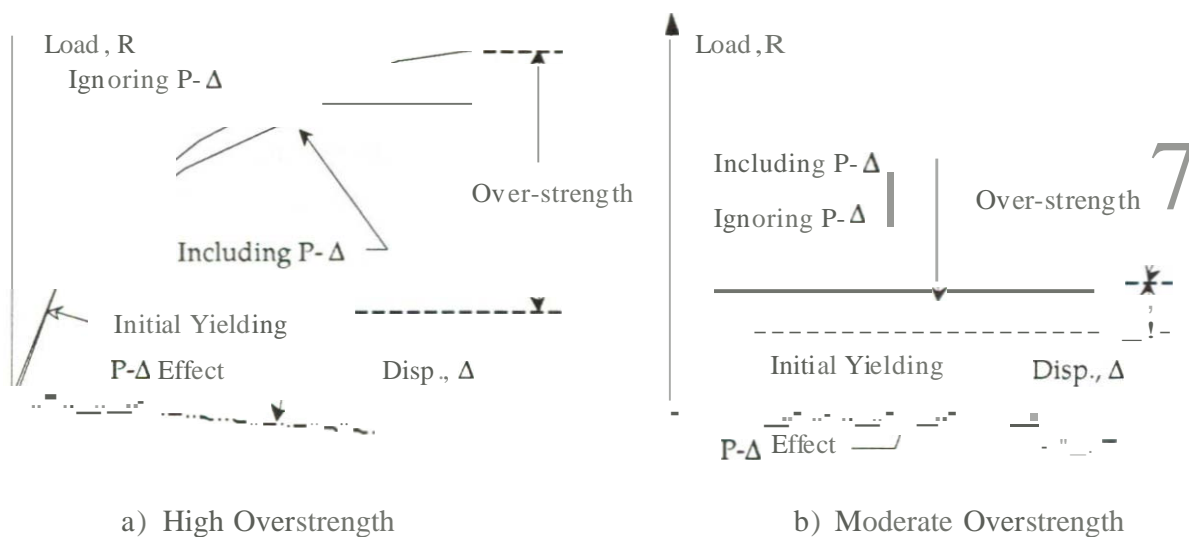


Fig. 8 $P-\Delta$ Effects on Actual Structures Exhibiting Deformation Hardening

It is also significant to note that most of the inelastic analyses of buildings discussed above used relatively short duration ground motions, like the Parkfield, El Centro, and Pacoima Dam records. Few studies considered ground motions with durations as long as those that might be anticipated in a magnitude 8 earthquake appropriate for design in California. Because of the sensitivity of $P-\Delta$ effects to the duration of shaking, some of the conclusions of these studies might change if longer duration events were considered.

Uniform Building Codeo -- The 1991 and 1988 editions of the Uniform Building Code (International Congress of Building Officials (1991 » contain explicit strength and stiffness provisions related to **P-Δ** effects. **P-Δ** effects need not be checked, if the displacements within a level do not exceed $0.02/R_w$ times the height of that level, R_w is a factor used in the UBC in the same way Z is used in the *Bridge Design Specification Manual*. For ductile moment frames, constructed of reinforced concrete, the factor R_w is equal to 12. It must be recognized that the loads considered for design in the UBC are at the working stress level, Thus, a better assessment of these provisions can be made if they are extended to the ultimate load level. Since a seismic load factor of 1.4 is utilized for reinforced concrete frames in the UBC, the elastic drift limit at ultimate load is $1.4(0.02)/12 = 0.0023$ or 0.23% of the height of the level in question.

In addition, the code contains an upper limit on elastic deflections under the design loads. These are " $0.04/R_w$ or 0.005 times the story height for structures having a fundamental period less than 0.7 secs. These drift limits are intended in part to help mitigate displacement induced damage to nonstructural components. For structures having a fundamental period of 0.7 secs. or greater, the calculated story drift shall not exceed $0.03/R_w$ or 0.004 times the story height." Again considering R_w equal to 12 and a load factor of 1.4, maximum elastic drift indices of 0.47% and 0.35% are obtained for a structure at ultimate load with periods below and above 0.7 secs, respectively.

While these values appear low, it must be recognized that the maximum inelastic drifts will be considerably larger, since an R_w value of 12 corresponds to a large value of Z ($\approx 12/1.4 = 8.6$). The corresponding limit on inelastic drifts (assuming $Z = 1$) is on the order of 3 and 4 % for structures with periods above and below 0.7 secs, respectively, assuming the elastic and inelastic displacements of structures are about equal. Similarly, the maximum drift index for not having to consider **P-Δ** effects becomes 2% for $Z = 1$. Notwithstanding these limits, the UBC permits even larger displacements provided one can demonstrate integrity is maintained for structural elements and for nonstructural components that affect life safety.

The UBC stipulates that **P-Δ** effects can also be ignored, if the following inequality holds:

$$P\Delta_w/L < 0.1V_w \quad (7)$$

in which V_w is the applied lateral working stress shear at a level; and
 Δ_w is the corresponding lateral deflection of the level,

The value of P includes contributions from unfactored dead and live loads. If the structure had a strength equal to $1.5V_w$ (assuming a load factor of **1.5** and a capacity reduction factor of 0.9) and the limiting deflection and force requirements in the code were met, **P- Δ** effects would reduce the effective capacity of the structure by $(=12(0.1)/1.5 = 0.8)$ 80% when the maximum inelastic deformation was developed. One reason such large reductions in capacity are permitted is because highly redundant frame structures are believed, as discussed above, to be 300 to 500% stronger than required by design. Significantly, this over-strength need not be demonstrated by a designer following the UBC provisions. If the inequality in Eq. 7 is violated, **P- Δ** effects are to be considered explicitly in design. Methods for doing this are not specified.

NEHRP Recommended Provisions. -- Recently recommended provisions for the development of building codes also contain provisions for handling **P- Δ** effects. These NEHRP provisions (BSSC (1988)) limit drifts to 1.5% of the height of the level under consideration (1% is stipulated for important structures that should be able to function immediately following an earthquake). Drifts are computed on the basis of magnified ultimate loads. In the case of ductile moment resisting frames, the ultimate load is obtained from the elastic design spectrum by dividing by 8. and the maximum inelastic displacements are then estimated by multiplying the displacements at the ultimate load by 5.5. If one considers displacements estimated using the full elastic response spectrum, the maximum drift limit would increase to $8(1.5\%)/5.5 = 2.2\%$. Assuming equivalent elastic spectra, the NEHRP provisions are considerably more restrictive than the those in the UBC.

P- Δ effects are evaluated at the ultimate load level rather than at the working stress level as done in the UBC. **P- Δ** effects can be ignored, if the following inequality is satisfied:

$$P\Delta_U/L < 0.1V_U \quad (8)$$

in which V_U is the applied lateral ultimate *design* shear at a level; and
 Δ_U is the elastic lateral deflection of the level corresponding to V_U .

Since elastic analysis is used to evaluate displacements, Eqs. 7 and 8 are equivalent. Where **P-Δ** effects need to be computed, a rational analysis to determine member forces is outlined, and the computed first order drifts are amplified by $(0.9/(1-(P\Delta_U/LV_U)))$. This is equivalent to applying additional lateral loads to the structure equal to $0.9P\Delta_U/L$, and is roughly equivalent to the UBC provisions.

New Zealand Standard Code of Practice. -- New Zealand seismic provisions (Pauley (1978), Standards Association of New Zealand (1976) are more restrictive than those in the U.S. when it comes to consideration of ductility, drift and **P-Δ** effects. For example, the maximum ductility allowed for ductile reinforced concrete frames is 4. The maximum inelastic drift is estimated by multiplying the (elastic) displacements at the design ultimate load by 2.5. In the lower stories of a multistory building the average estimated drifts over the height of the structure are doubled to account for the probable deflected shape of a frame structure and the tendency of yielding to concentrate in the lower stories. The drifts computed in this fashion cannot exceed 1% of the story height. These provisions are much more restrictive than U.S. provisions. If the drifts are extended to the level corresponding to the elastic design spectrum, a drift index of only 1.6% would be permitted in comparison to 2.2% for the NEHRP provisions and 3 to 4% for the UBC.

According to the New Zealand standards **P-Δ** effects are to be evaluated, if the following inequality is violated:

$$\lambda P\Delta_U/H < 0.15 V_U \quad (9)$$

in which H is the total height of the structure;
 V_U is the ultimate design shear at a level;
 Δ_U is the ultimate displacement at the roof; and
 λ is a displacement amplification factor accounting for the inelastic displaced shape of a frame structure.

For the lower levels of multistory structures λ is taken as 2, but for single level structures λ would equal unity. The value of Δ_U is significantly different in this expression than for Y_u in the NEHRP provisions. Here, it is taken to be 2.5 times larger than the deflections computed elastically for the ultimate design loads. Recognizing this fact, the New Zealand provisions become 67% more restrictive than NEHRP provisions. Moreover, where **P-Δ**

effects are to be computed, the magnitude of the incremental design force is approximately 2.5 times that suggested by the NEHRP provisions (for all other factors being equivalent). It can be shown that the New Zealand provisions are based on Eq. 1 with C equal to 2; i.e., conservation of energy under monotonic loading is assumed (Pauley (1977)).

Special Provisions for Inverted Pendulum Structures. -- The NEHRP provisions contain special requirements for systems called inverted pendulums. In these structures, the lateral load resisting system resists the seismic forces essentially through cantilever action as well as provides vertical support for gravity loads. Although building code provisions were not formulated with bridge structures in mind, this system corresponds to single column viaducts or to multi-column viaducts with a pin provided at one end of the columns. Similarly, the designation might be applied to multiple level viaducts which have pins provided at one end of all columns (as envisioned in the San Francisco area double deck viaducts).

For ductility detailed members in steel or concrete, the NEHRP provisions reduce the elastic response spectrum by a factor of 2.5 (rather than 8). This results in lateral forces 4.8 times greater than those used for the design of frames. Moreover, the drift limits imposed on inverted pendulum structures are computed at the full elastic response level, and not at a reduced level as is the case for framed structures. This results in a maximum permitted drift index of 1.5% for the NEHRP provisions (69% of the equivalent value permitted for framed structures).

The more stringent requirements for inverted pendulum structures is a result of the reduced redundancy of these structures, their limited post-yield over-strength relative to frames, and the formation of plastic hinges in critical vertical load carrying elements (columns).

Earlier editions of the UBC required forces for inverted pendulum structures approximately 3 times those used for ductile moment frames. However, the UBC no longer contains explicit provisions for inverted pendulum structures. The frame provisions in the UBC described above are not applicable to the design of viaducts, since plastic hinges are allowed to develop in bridge columns (column hinges are generally not permitted in reinforced concrete buildings above the foundation). The UBC would currently categorize bridge structures as "undefined systems." For these systems, dynamic analysis is required to determine the level of elastic level forces and displacements, and technical and

experimental data must be used to substantiate the force reduction factors utilized in design. No special drift limitations are stipulated for undefined systems.

Comments. -- Caltrans provisions for determining $P-\Delta$ induced forces for the Terminal Separator are slightly more stringent than New Zealand provisions for building design. They go far beyond what is currently done in the U.S. for buildings. The loose restriction on lateral displacements in the Terminal Separator criteria may result in situations where $P-\Delta$ effects may become substantial. On the other hand, the maximum limit of 1% of the story height (computed for $Z = 1$ and considering cracked section properties) as used for the retrofits of the San Francisco double deck viaducts is more restrictive than New Zealand provisions. However, this restriction may be warranted because of the tendency of these types of structures to form "soft stories." In this case yielding would concentrate in one level and the other would remain essentially elastic. This type of behavior would substantially increase (possibly double) the inelastic displacement demand on the yielding level relative to the that computed on an elastic basis (see Fig. 9). Caution must be exercised in the design of systems susceptible to soft or weak stories because of these increased local ductility demands.

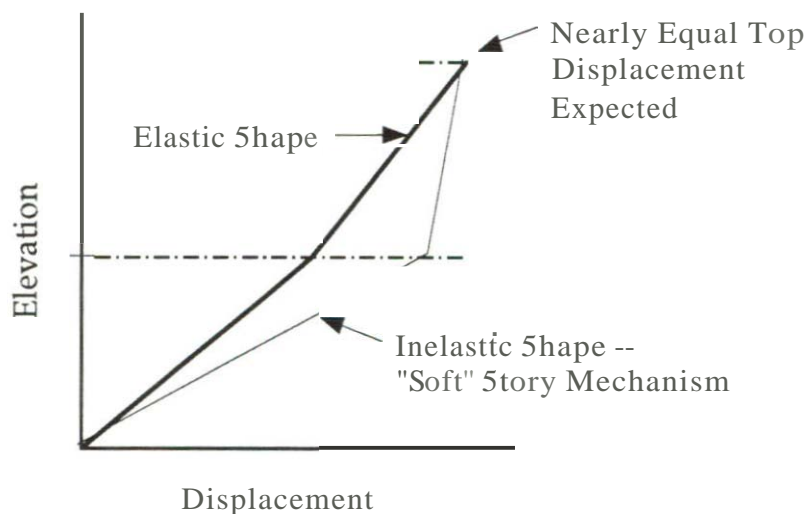


Fig. 9 Soft Story Formation in a Double Deck Viaduct

The studies on $P-\Delta$ effects that have been conducted to date are limited in scope and applicability. For instance, few consider relevantly long ground motion durations, realistic restoring force characteristics for reinforced concrete, and idealizations applicable to bridge

structures. Most studies are formulated in a manner differing from that used in design. Additional studies are needed to clarify these issues for the design of bridges.

ANALYTICAL INVESTIGATION OF DESIGN METHODS

To assess design methods being considered by Caltrans for some of its major structures, an analytical study was carried out using simple single degree of freedom models (like Fig.1). Most previous analytical investigations non-dimensionalize the equation of motion for the single degree of freedom model, and thereby study the time to collapse or the increase in ductility demand as a function of parameters that have no direct design interpretation.

In this study, an attempt is made to formulate the analysis problem in a design context. The particular focus of these analyses is identification of the strength increase needed in order for a structure subjected to geometric nonlinearities to develop the same displacement ductility it would if the geometric nonlinearities did not exist. In this way, the severity of inelastic response for the two systems would be equivalent.

A series of simple structures were designed elastically. To do this, Z reduction factors were applied directly to the response spectra for the specific earthquake records used in the analyses (thereby avoiding problems associated with scaling ground motions so that their spectrum would match some smoothed target spectrum). The ductility demands on these systems were then numerically computed ignoring $P-\Delta$ effects. $P-\Delta$ effects were then introduced and the strength of the structure was increased incrementally until the ductility demand of this system equaled that of the structure designed ignoring $P-\Delta$ effects. In this fashion, the inelastic response of the structure would be no more severe than would be permitted in conventional design. Comparison of the required strength increases with those indicated by Eqs. 1 and 2 provides an insight into the reliability of these equations and may suggest improvements, where needed.

In this section, the analytical models and ground motions used are first described. A brief series of time history responses are then presented to illustrate some of the basic behavior exhibited by inelastic systems subjected to $P-\Delta$ effects. The results of the parameter study are then examined to assess the effect of period, intensity of geometric nonlinearity, and Z factor on required strength increases. In addition to considering different models for the restoring force characteristics of the structure, the influences of ground motion type and duration are also partially assessed. The suitability of Eq. 1 is addressed, and an alternate method is suggested.

Analytical Model. -- To carry out these investigations, a series of single degree of freedom models with nonlinear restoring force characteristics are subjected to time history analyses. Two types of hysteretic models are considered. The first is a bilinear hysteretic model with a zero or negative post-yield tangent stiffness, as shown in Figs. 3 through 6. The second model is a conventional stiffness degrading model with $P-\Delta$ effects idealized as shown in Fig. 7.

These models must be considered to represent the "effective" mechanical characteristics of the structure, and not just those based on the *Bridge Design Specification Manual*. Most of the analyses at this time are based on the bilinear idealization. Initial discussions thus focus on results for the classic bilinear model. Results for stiffness degrading models are discussed subsequently,

For simplicity, only four structural periods are considered: 0.5, 1.0, 2.0 and 3.0 secs. These periods will be shifted in the analyses as required to account for $P-\Delta$ effects, but they will be referred to in the text by their nominal values. For firm ground conditions computed elastic and inelastic displacements are expected to be nearly equal for all structure periods considered (Newmark (1971) and Mahin (1981)). For ground motions recorded at soft soil sites, inelastic displacements computed may exceed elastic values for structures with periods less than the predominant period of the ground.

Viscous damping in all analyses was taken to be 5% of critical. Hysteretic energy loss associated with material nonlinearity is accounted for automatically in the analyses.

Ground Motions. -- Only two ground motions are considered in these preliminary studies. The first is the S69E component of the 1952 Taft (Kern County) earthquake. This 7.7 event was recorded on 40 ft. of alluvium at an epicentral distance of 41 km. The second record is the outer wharf record obtained in Oakland during the 1989 Loma Prieta earthquake. This soft soil site is approximately 70 km from the epicenter of this magnitude 7.1 earthquake. To assess the effects of larger and longer duration events, a copy of the Taft record has been added in some of the analyses to the end of the original record, effectively doubling the duration of the earthquake.

Time histories and response spectra for these records are shown in Figs. 10 and 11. The predominant periods in the Taft spectra are near 0.3 secs., well below the range of periods considered herein. The Oakland Wharf record has predominant periods around 0.7

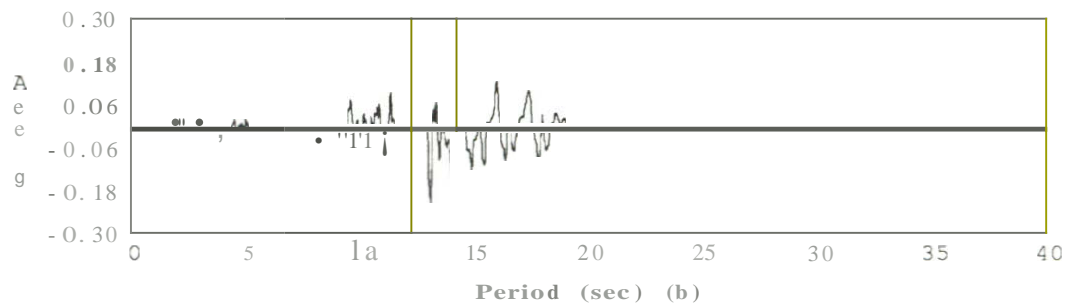
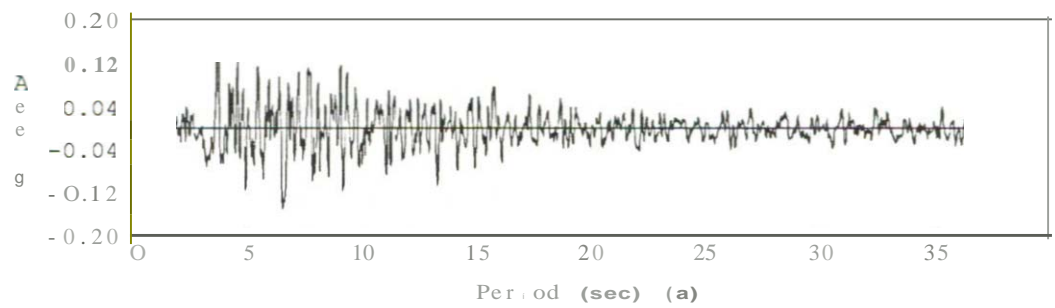


Fig. 10 Earthquake Records, a) Taft 569E, 1952 Kern County. b) Oakland Wharf 305 Deg, 1989 Loma Prieta

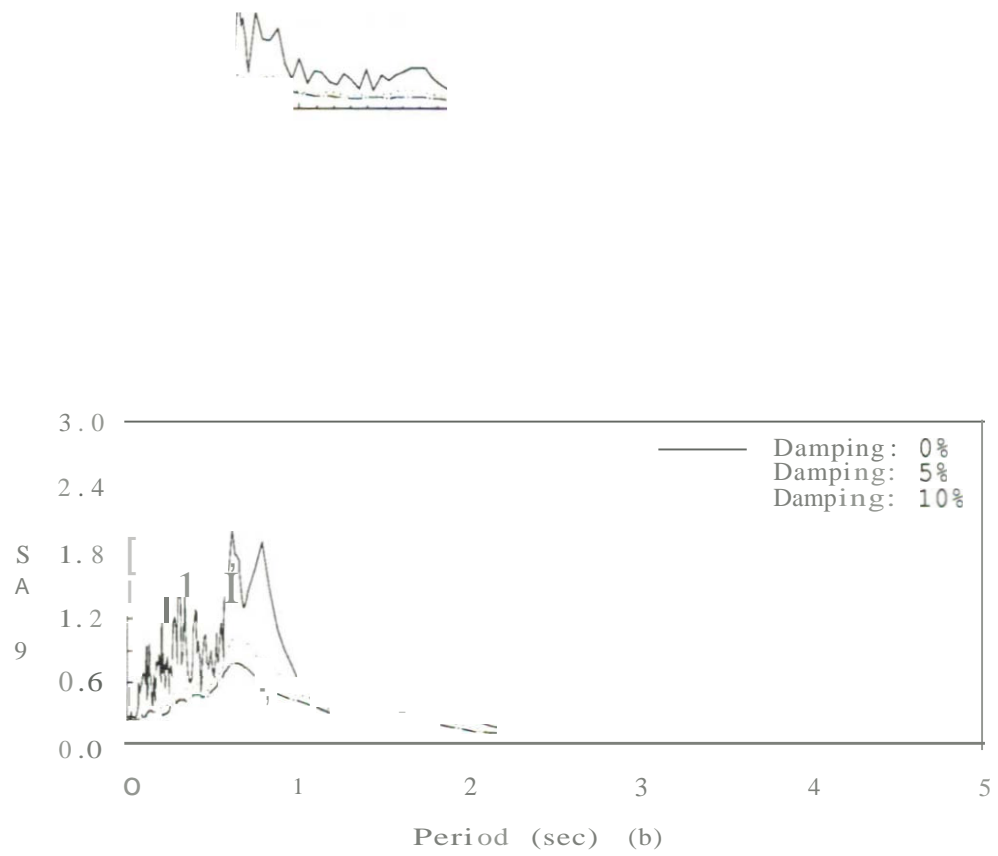


Fig. 11 Earthquake Records Linear Spectra. a) Taft 569E. 1952 Kern County. b) Oakland Wharf 305 Deg. 1989 Loma Prieta

and 1.6 secs. As a result, one would anticipate that ductility demands could substantially exceed the 'effective' Z values for the structures with 0.5 and 1.0 sec periods.

Design of the Basic Structures Analyzed. -- The strengths of the structures analyzed were determined from the elastic response spectrum for the earthquake records used in the analysis. The plastic strength of the structure was computed, as done in current design procedures, by dividing the design spectral acceleration (for the appropriate damping and period) by an 'effective' ductility and importance factor, Z. Rather than using a smoothed ARS spectrum in these computations, the spectrum for the particular earthquake used in the analysis is employed. This avoids problems associated with fitting an earthquake's spectrum to a predetermined smoothed spectrum.

The yield strength of a system ignoring **P-Δ** effects would be:

$$R_y = m \cdot S_a / Z \quad (10)$$

in which m is the mass of the system (here equal P/g); and
 S_a is the pseudo-spectral acceleration for the earthquake record, period and damping considered.

In these analyses, effective values of Z were taken to be 1, 2, 4, 6, and 8. The factor Z is taken to be independent of period, unlike the *Bridge Design Specification Manual* where Z varies (at least in the short and intermediate period ranges) inversely with period. These Z values should be taken to be "effective" values, since a structure designed for a given yield level will be stronger due to over-design, real material yield properties, and strain hardening. Thus, the 'effective' Z values used herein do not correspond to current Caltrans design practices. The 'effective' Z value appropriate for a simple bridge designed with a Z of 4 may be between 2 and 3 because of the inherent over-strength.

If the structure remained elastic, the maximum elastic base shear, **R_e**, would be:

$$R_e = m \cdot S_a \quad (11)$$

and the maximum elastic displacement would be:

$$\Delta_e = S_d \quad (12)$$

where S_d is the spectral displacement corresponding to S_{ao}

As shown in Fig. 12, several useful relationships can be developed based on the slope of the post-yield branch of the envelop curve. Here, the slope $-P/\Delta$ is represented as a fraction p times the initial elastic stiffness of the structure. Rather than setting the value of p to arbitrarily selected values (e.g., 0.02) as done in most other studies (e.g., Bernal (1987)), p will be selected herein to reflect the severity of the $P-\Delta$ loads at the elastic design displacement corresponding to $Z = 1$.

For intermediate and long period structures it is usual in the absence of $P-\Delta$ effects to assume that the maximum elastic and inelastic deflections are the same. If this is done here, it is possible to estimate the reduction in apparent strength that results from the $P-\Delta$ effects (i.e., $P\Delta_e/L$). This reduction, at the elastic displacement Δ_e , is represented in Fig.12 by R_y/α . The structural strength remaining at this point to resist seismic actions is given by:

$$R_y'' = R_y (1 - 1/\alpha) \quad (13)$$

It is believed that this type of calculation lends itself to current elastic design methods, and that a designer may already compute R_y'' in order to assess intuitively the likely severity of $P-\Delta$ effects,

In this study, values of α are taken as 2, 4, and 8 representing reductions in strength at the maximum elastic displacement of $1/2$, $1/4$ and $1/8$, respectively, of the strength computed disregarding $P-\Delta$ effects. While the α value of 2 may seem severe, it should be recognized that the UBC relation (Eq. 8) permits $P-\Delta$ effects to be ignored for α values as low as 1.25! This latter value corresponds to an 80% loss in capacity, provided the structure has negligible over-strength,

From geometry, an expression can be obtained for p in terms of Z and α :

$$p = 1/(\alpha Z) \quad (14)$$

Values of p corresponding to the conditions considered in this parameter study are shown in Table 1 below.

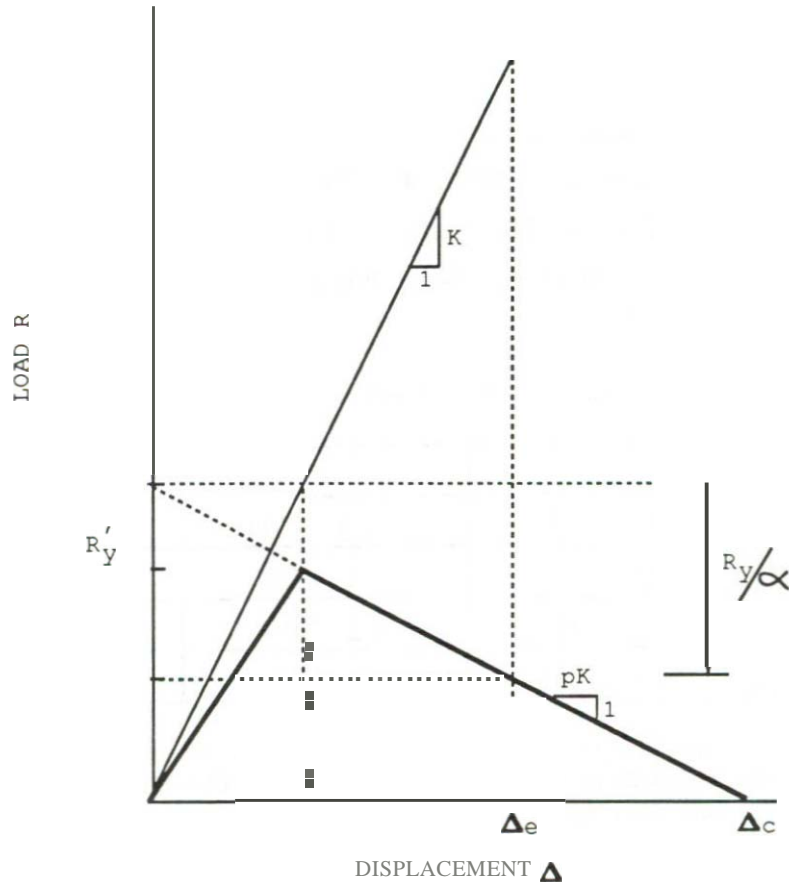


Fig. 12 P- Δ effects on load-resistance curves.

The reduced stiffness of the structure, K' , in the elastic range due to $P-\Delta$ effects, may be written as:

$$K' = K (1 - p) \quad (15)$$

and the corresponding shifted elastic period, T' , is given by:

$$T' = T / (1 - p)^{0.5} \quad (16)$$

As can be seen in Table 1, the elongation of period due to geometric nonlinearities is less than **5%** for most of the structures considered. However, in the most severe case (low α and Z) a shift of **15%** would occur. These elongated periods can significantly alter the dynamic response of a structure and change the required design force if the shifted periods were considered in design.

Z	α		
	2	4	8
2	-25.0 (1.15)	-12.5 (1.07)	-6.3 (1.03)
4	-12.5 (1.07)	-6.3 (1.03)	-3.1 (1.02)
6	-8.3 (1.04)	-4.2 (1.02)	-2.1 (1.01)
8	-6.3 (1.03)	-3.1 (1.02)	-1.6 (1.01)

Table 1 Post-yield Tangent Stiffness as a Percentage, p , of the Initial Elastic Stiffness for Parameters Considered in this Study
(Values in Brackets Represent Change in Elastic Range Period as Fraction of Structural Period Computed Disregarding Geometric Nonlinearities)

The yield displacement of the structure, $\Delta_y = S_d/Z$, does not change with the application of geometric nonlinearities. However, the apparent yield strength, R'_y , does reduce to:

$$R'_y = K' \Delta_y = K (1 - p) \Delta_y = K S_d/Z \quad (17)$$

Continued loading in the inelastic range will gradually decrease the strength of the structure until the forces induced by the geometric nonlinearities cancel the mechanical resistance of the structure. At this point the structure is statically unstable and will collapse

under its own weight. The critical displacement at which this will occur is easily computed from geometry (or statics: i.e., $P\Delta/L = R_y$) and can be expressed as:

$$\Delta_c = Sd/(pZ) = \Delta_y/p \quad (18a)$$

or

$$\Delta_c = \alpha \Delta_y Z = \alpha S_d \quad (18b)$$

and the corresponding static collapse displacement ductility is given by:

$$\mu_c = \Delta_c/\Delta_y = 1/p = \alpha Z \quad (19)$$

Since μ_c corresponds to static buckling of the structure, it is clear that the design ductility value should be far smaller. In fact, Takizawa and Jennings (1980) noted that the dynamic stability limit may be less than the static one by as much as 20 to 70%, depending on the ground motion and structural characteristics.

From Eq. 18b and the assumption that for long period structures (Newmark (1971)) Z equals the structure's design ductility factor, α is the factor of safety against overall static buckling of the structure. Thus, for the examples considered herein, the safety factors range from 2 to 8. Given the large scatter in response associated with inelastic dynamic behavior, these factors of safety may prove inadequate for small α .

It should be noted that the residual capacity of a structure should be maintained above a level necessary to prevent collapse during aftershocks. For example, if large permanent offsets occur, the strength of the structure following an earthquake will be significantly reduced by the $P-\Delta$ effects. This reduced capacity may not be sufficient to maintain stability in the many aftershocks anticipated following a major earthquake. Many structures have been observed to fail during aftershocks as a result of accumulated inelastic deformations (Mahin (1976)).

This problem will not be addressed herein. However, special design procedures may need to be developed in order to limit displacements to insure satisfactory behavior during aftershocks, or immediate post-earthquake shoring should be planned. Preliminary studies in this region and elsewhere (MacRae and Kawashima, 1991) indicate that the residual displacements in structures subjected to even moderate geometric nonlinearities are nearly equal to the maximum displacements developed during the earthquake. As such,

simple procedures may be devised to estimate the residual capacity of the structure (i.e., $R_y \text{ residual} \approx R_y'' = R_y - P\Delta_{\max}/L$ assuming that Δ_{\max} can be accurately estimated). Similar procedures have been outlined by Berna! (1987).

Response of Basic Structures. -- The structures analyzed were designed considering them to have a unit mass, one of the pre-selected periods and Z values, and strengths given by Eq. 10 based on the actual response spectrum for the record in question for 5% viscous damping. The nonlinear analyses were carried out using the program NONSPEC (Lin and Mahin (1983)).

The displacement ductility requirements for both earthquakes are shown in Fig. 13 considering effective Z factors of 1, 2, 4, 6 and 8. The systems considered in these plots do not include geometric nonlinearities. The difference between the maximum displacement ductility computed and the Z value used in design is an indication of how well elastic displacements predict inelastic values.

For the period range considered, ductility factors resulting from the Taft record are not highly sensitive to period. For low Z values, the value of Z reasonably approximates the displacement ductility values. However, as Z increases, the displacement ductility becomes as much as 75% larger than Z .

For the Oakland Wharf record the ductility demands are sensitive to period. Ductilities computed for the short period structures are much higher than Z as expected, sometimes by as much as 340%. The same trend would be expected for the Taft record, if periods less than 0.3 seconds were considered (Mahin (1981)).

The ductility and importance factor Z used in the *Bridge Design Specification Manual* increases in the short period range. Unusually large ductilities might thus be expected for bridges with periods shorter than the predominant period of the design spectrum since ductility demands tend to exceed Z as Z becomes large and as the period of the structure falls below the predominant period of the ground motion.

The consequences of adding geometric nonlinearities to these systems can be seen in Fig. 14. The hysteretic response of an elasto-perfectly plastic system with Z equal to 4 and a period of 2.0 seconds shown for the Taft record in Fig. 14a. As indicated in Fig. 13a,

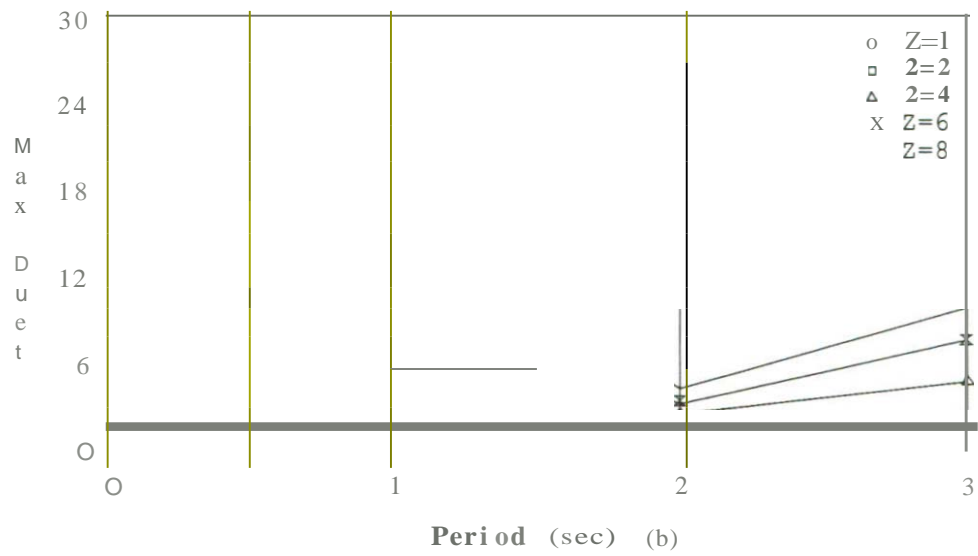
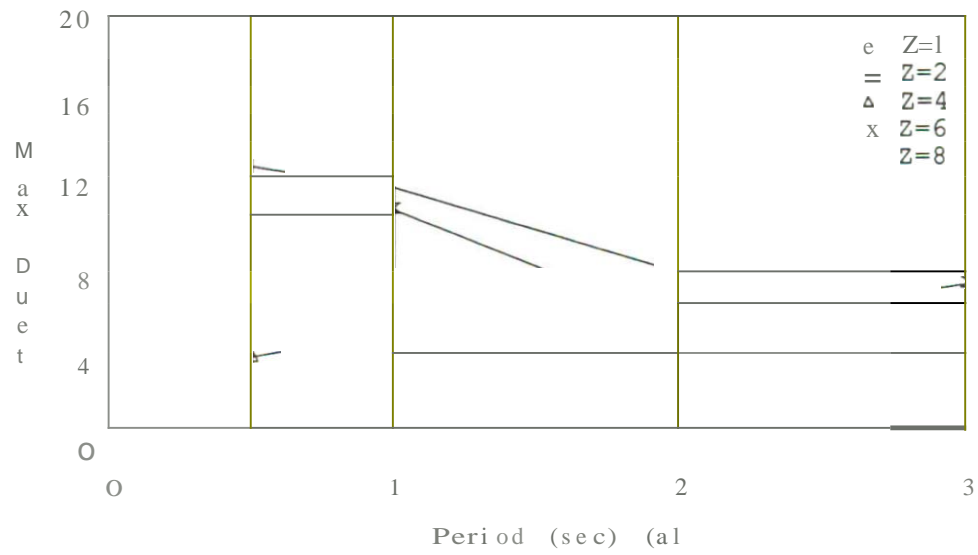


Fig. 13 Durtility demand for design reduction factor. a) Taft 569E, 1952 Kern County. b) Oakland Wharf 305 Deg. 1989 Loma Prieta

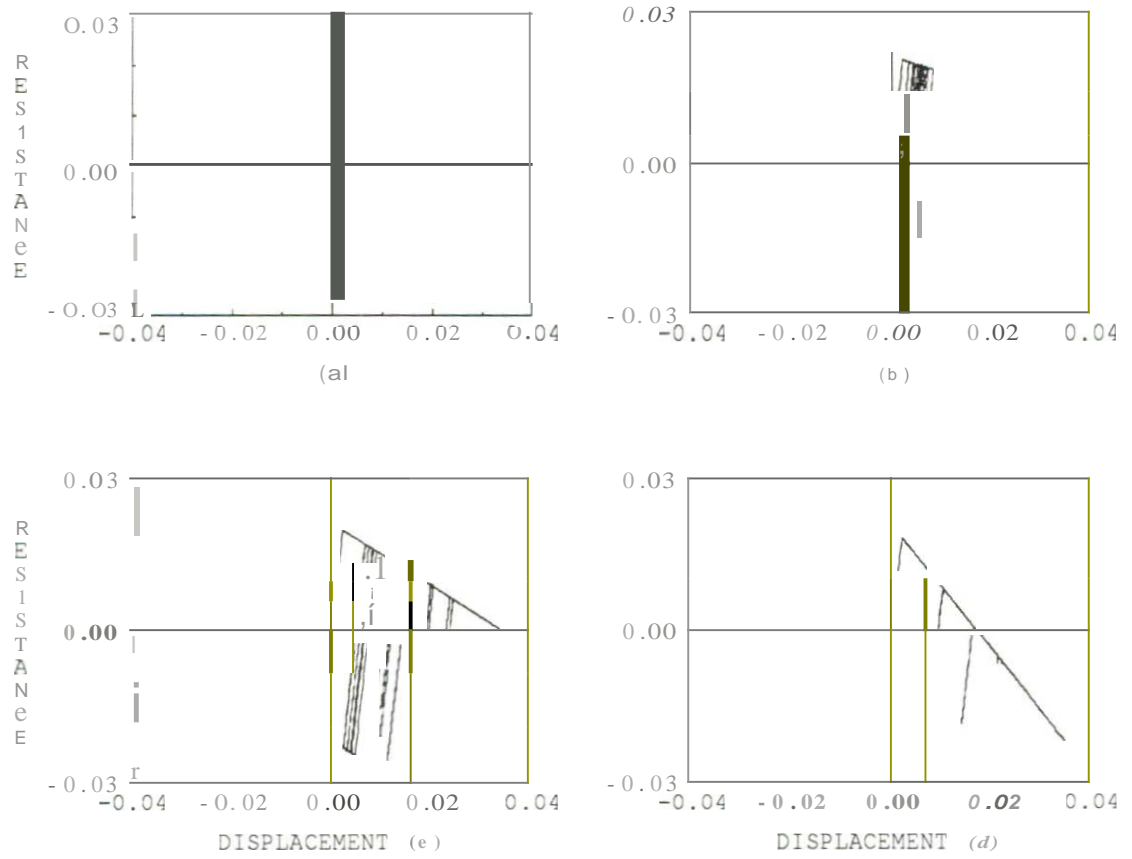


Fig. 14 Hysteresis loops for Elasto-Perfectly Plastic Models (EPP). Taft 569E. 1952 Kern County. Period 2 sec., $Z=4$. a) No $P-\Delta$. b) $P-\Delta$ $\alpha=8$. c) $P-\Delta$ $\alpha=4$. d) $P-\Delta$ $\alpha=2$.

the ductility demand for this structure nearly equals Z . When geometric nonlinearities are introduced, the displacement demands increase as shown in Figs. 14b, 14c and 14d (for reduction in strength at maximum elastic displacement of 1/8, 1/4 and 1/2 of the initial strength of the structure, respectively). It can be seen that *unstable* response results for c 's of 2 and 4.

The maximum displacement ductility demands for elasto-plastic systems with and without $P-\Delta$ effects are listed in Table 2 and 3 for the Taft and Oakland Wharf records, respectively. In the tables, an α value of infinity represents the case without geometric nonlinearities. It can be seen from the tables that most of the cases with α equal to 2 collapse and many ductility demands for α equal to 4 exceed μ_c .

The ratio of displacement demands for systems with and without geometric nonlinearities are plotted in Figs. 15 and 16. From this data it is clear that the effect of geometric nonlinearity is particularly severe for systems with large α (P/L) and Z (ductility) ratios. In many cases, the ductility demands exceed the collapse ductility capacity given by Eq. 19. In some cases there were minor reductions in ductility demand for structures with geometric nonlinearities included. This reduction is likely due to the geometric nonlinearities shifting the period of the structure (see Table 1) away from a local peak in the spectrum for the particular earthquake used in the analysis. However, the maximum displacements of systems with $P-\Delta$ effects included were generally much greater than estimated by $Z \cdot \Delta_y$. For the Taft record 81%, 88 % and 100 % of the structures considered have increases in ductility demands greater than 20% when α is taken to be 8, 4 and 2, respectively. It can be seen that $P-\Delta$ effects cannot be safely ignored, except for the very restrictive case of $\alpha \geq 8$ and $Z \leq 2$.

Required Strength to Limit Increase in Ductility Demands due to $P-\Delta$ Effects. -- To mitigate the adverse effects of geometric nonlinearities on ductility demands, one can increase either the structure's stiffness or strength, or both. As discussed in the section on design procedures, it may be most effective to control $P-\Delta$ effects by limiting drifts. Reducing drifts has the effect of reducing the maximum $P-\Delta$ induced forces, and results in an increased α for all other factors being the same. However, it may not be practicable to increase stiffness substantially in many situations. If stiffness cannot be increased, one must establish the required strength increment to limit the ductility demand on the system to acceptable levels.

Z	Period or Collapse Ductility, μ_c		α			
			2	4	8	∞
2	μ_c		4	8	16	∞
	Period	0.5	∞	2.49 (-12)	3.47 (22)	2.84
		1.0	∞	∞	4.05 (78)	2.27
		2.0	∞	1.81 (14)	1.64 (3)	1.59
		3.0	∞	2.49 (74)	1.55 (8)	1.43
4	μ_c		8	16	32	∞
	Period	0.5	∞	∞	27.5 (526)	4.39
		1.0	∞	∞	15.2 (176)	5.50
		2.0	∞	∞	3.72 (1)	3.67
		3.0	∞	∞	7.06 (65)	4.28
6	μ_c		12	24	48	∞
	Period	0.5	∞	∞	∞	10.5
		1.0	∞	∞	∞	11.1
		2.0	∞	∞	8.54 (43)	5.96
		3.0	∞	∞	14.7 (90)	7.72
8	μ_c		16	32	64	∞
	Period	0.5	∞	∞	∞	13.2
		1.0	∞	∞	55.6 (355)	12.2
		2.0	∞	∞	16.0 (86)	8.62
		3.0	∞	∞	17.6 (80)	9.77

Table 2 Displacement Ductility Demands for Taft Earthquake Record; Elasto-plastic Systems: Values in Brackets indicate Percentage Change from Case with $\alpha = \infty$,

Z	Period or Collapse Ductility, μ_c		α			
			2	4	8	∞
2	μ_c		4	8	16	∞
	Period	0.5	∞	∞	2.85	2.72
		1.0	∞	2.99	2.65	2.38
		2.0	1.48	1.52	1.53	1.48
		3.0	∞	2.78	2.20	2.07
4	μ_c		8	16	32	∞
	Period	0.5	∞	∞	27.6	10.1
		1.0	∞	∞	6.14	5.86
		2.0	2.96	2.13	2.32	2.44
		3.0	∞	∞	6.47	4.63
6	μ_c		12	24	48	∞
	Period	0.5	∞	∞	∞	20.2
		1.0	∞	∞	18.5	11.5
		2.0	∞	3.45	2.84	3.05
		3.0	∞	21.0	10.5	7.54
8	μ_c		16	32	64	∞
	Period	0.5	∞	21.3	∞	26.8
		1.0	∞	∞	25.6	16.2
		2.0	10.1	4.28	2.49	4.11
		3.0	∞	28.5	13.9	9.75

Table 3 Displacement Ductility Demands for Oakland Wharf Record;
Elasto-plastic Systems

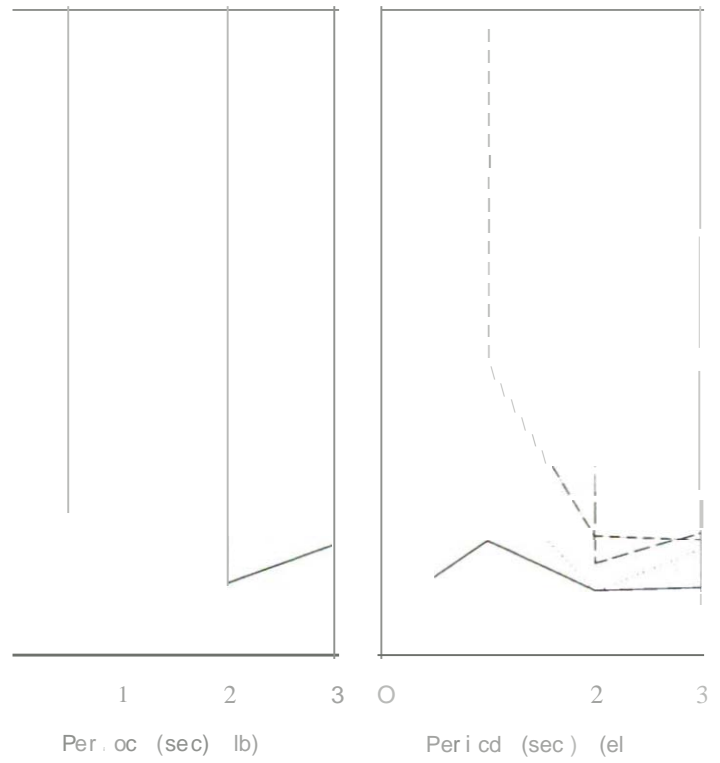


Fig. 15 Ratio of Displacement for $P-\Delta$ and non $P-\Delta$ EPP models. Taft 569 E. 1952 Kern County. a) $\phi = 2$. b) $\phi = 4$. c) $\phi = 8$.

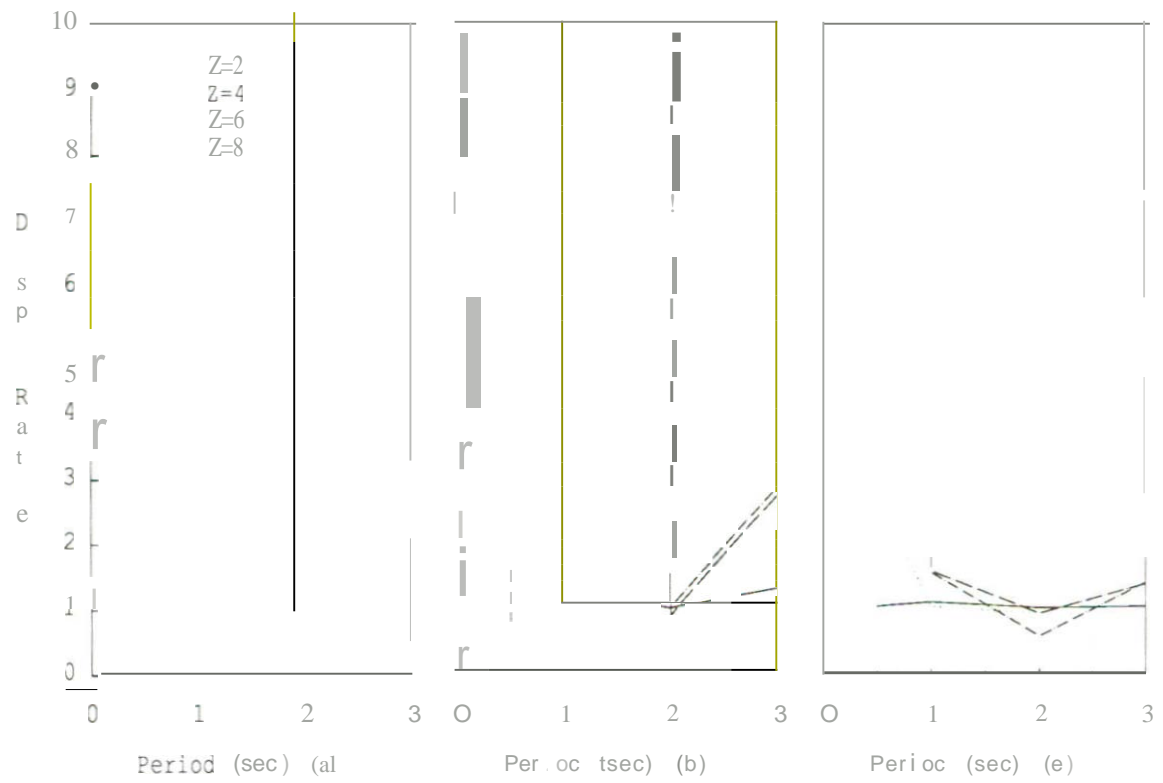


Fig. 16 Ratio of Displacement for p. Δ and non p. Δ EPP models. Oakland Wharf 305 Deg. 1989 Loma Prieta. a) $\alpha = 2$. b) $\alpha = 4$. e) $\alpha = 8$.

For design purposes, Eq. 1 might be used to establish the required strength. However, the appropriate value of \mathbf{C} has yet to be determined. In this phase of the investigation, the strengths of the structures previously analyzed are increased. The amount of increase is selected so that the ductility demand on the strengthened structure with $\mathbf{P-\Delta}$ effects applied is essentially identical to that for the original structure analyzed disregarding $\mathbf{P-\Delta}$ effects. If the inelastic response without $\mathbf{P-\Delta}$ effects was acceptable, then the response of the strengthened system should be acceptable as well.

It is important to realize, however, that the displacements of the original and strengthened systems are not the same even though the ductility demands are. This is because strengthening will increase the yield displacement, resulting in larger total lateral displacements for the strengthened structure at the same ductility demand. This is shown schematically in Fig. 17. This may have an important effect where displacements need to be controlled to limit pounding, discomfort to riders, repair costs, and so on. In such cases even more restrictive design criteria may be required.

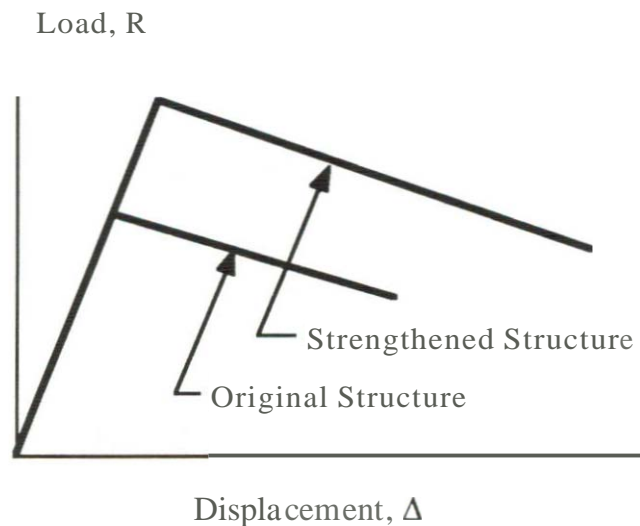


Fig. 17 Effect of Strengthening on Maximum Displacement for Structures with Same Ductility

The effect of strengthening on dynamic response can be seen in Fig. 18 where a structure with a period of 2.0 sec and designed with \mathbf{Z} equal to 4 is subjected to the Taft earthquake record. In Fig. 18(a) the elastic force and shear demands are plotted as well as the basic envelop of the reduction in strength that might occur if $\mathbf{P-\Delta}$ effects were considered. In Fig. 18(b) inclusion of $\mathbf{P-\Delta}$ effects ($\mathbf{a} = 2$) is seen to lead to collapse of the

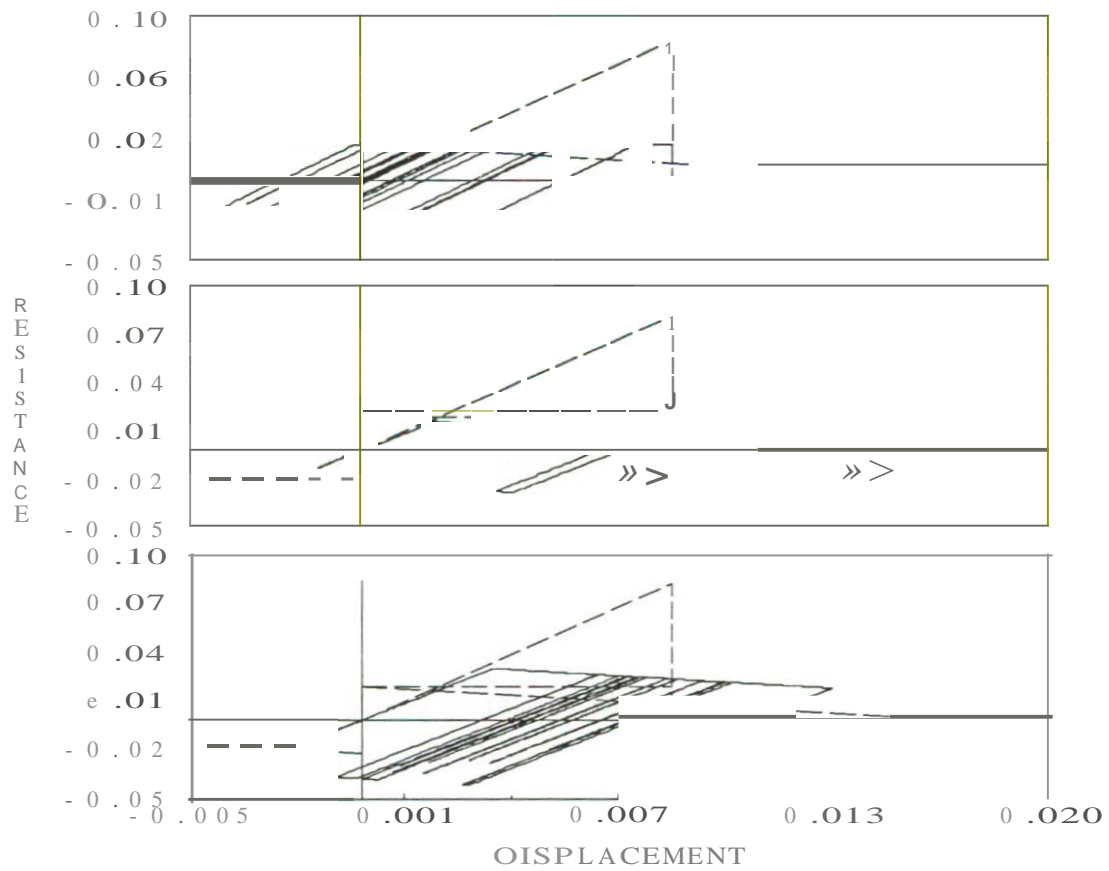


Fig. 18 Elasto Perfectly Plastic Models Hysteresis Loops. Taft SG9E. 1952 Kern County. Period 2 sec, $Z = 4$. a) No P- Δ . b) $0=2$. $\gamma = 1$. c) $0=2$. $\gamma = 1.76$.

system. An increase in strength equal to 76 % of the original design strength is seen to avoid collapse in Fig. 18(c). Although the displacement demand is much larger in this latter case, the ductility demand is identical to the case where **P-Δ** effects are ignored.

It is important to note in this example that, while the reduction in strength at the elastic displacement of the structure is 50 % of the initial yield strength ($\alpha = 2$), the required increase in strength to achieve the same ductility demand is 76 % of the initial yield strength. The required strengthening exceeds the magnitude of the **P-Δ** effect at the elastic deflection by a factor of 1.5. Thus, it may not be possible to obtain reliable or conservative design forces from Eq. 2 (with the maximum displacement estimated by the maximum elastic displacement of the system) or from Eq. 1.

This same process is repeated for all of the structures considered in the previous parameter study. The increase in strength used to compensate for **P-Δ** effects (Eqs. 1 and 2) may be expressed in terms of the parameters introduced above. In this case, it will be assumed that an arbitrary fraction, β , of the total strength loss at the maximum elastic displacement will be added to the strength of the system. Since it is expected that design will be done on the basis of elastic analysis, the maximum displacements are estimated using the maximum elastic displacements of the structure, Δ_e , computed ignoring **P-Δ** effects.

The total required strength of the system, R^*_y , is given, as shown in Fig. 19, by the expressions:

$$R^*_y = R_y + R_\Delta = R_y + \beta R_y / \alpha = R_y (1 + \beta / \alpha) = \gamma \cdot R_y \quad (20a)$$

or

$$\gamma = (1 + \beta / \alpha) \quad (20b)$$

For this formulation the value of β can be determined empirically to achieve the desired equality of ductilities in systems with and without **P-Δ** effects. The case where β equals unity corresponds directly to Eq. 2 provided $\Delta_{max} = \Delta_e$. It is useful to use the same formulation to assess Eq. 1. A general relation between β and θ (from Eq. 1) can be established assuming that the maximum inelastic displacement is $\Delta_y Z$ (i.e., $Z = \mu_\Delta$). The relation may be developed considering:

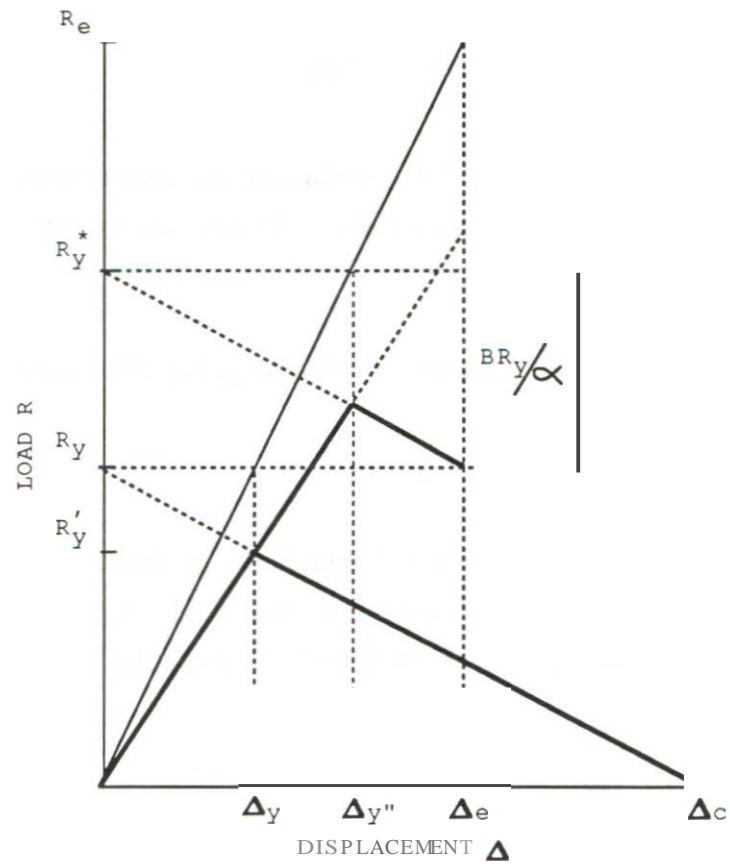


Fig. 19 Strength increase for P-Δ effects.

$$R_{\Delta} = M_{\Delta} / L = [1 + \mu_{\Delta}] P \Delta_y / (\theta L) = \beta R_y / \alpha \quad (21)$$

If we assume $S_d = \mu_{\Delta} \Delta_y$, $P \mu_{\Delta} \Delta_y / (L) = R_y / \alpha$. Consequently, from Eq. 21:

$$P \Delta_y / (\theta L) + R_y / (e a) = \beta R_y / \alpha \quad (22)$$

which, by noting that $R_y L / (\alpha P \Delta_y) = Z$ (from geometry considerations in Fig. 12), can be solved for e :

$$\theta = [1 + \alpha P \Delta_y / (L R_y)] / \beta = [1 + \alpha Z] / \beta \quad (23)$$

For 100% ($\beta=1$) of the **P-Δ** effect at the maximum elastic displacement to be considered for $Z = 4$, the value of θ must be taken equal to 1.25, as noted earlier. From Eq. 23, $\theta = 2.5$ for $\beta=1/2$ and $Z = 4$.

From a design perspective it is useful to rearrange Eq. 23 to solve for β :

$$\beta = [1 + \alpha Z] / \theta \quad (24)$$

so that for $\theta = 2$ and $Z = 4$, $\beta = 5/8 = 0.63$. Similarly, for the design values used on the Terminal Separator ($e = 1.5$ and $Z = 4$) $\beta = 5/6 = 0.83$. That is, the strength of the structure must be increased by 83% of the **P-Δ** forces computed at the maximum elastic displacements, S_d .

The total design strength required can be expressed, using Eqs. 20 and 24, in terms of the increase in original design strength:

$$\gamma = R^* Y / R_y = 1 + [1 + \alpha Z] / (e a) \quad (25)$$

Using this equation, for $Z = 4$ and $\theta = 1.5$, Y values of 1.42, 1.21 and 1.10 are obtained for a values of 2, 4 and 8, respectively. It must be recognized that **P-Δ** effects will reduce the actual capacity of the structure from these values (see Eq. 17).

The formulation of Eq. 25 in terms of θ implies that the required increase in strength to compensate for **P-Δ** effects decreases with increasing Z . This is counter-

intuitive and results from the "1" term inside the brackets in Eq. 1. The γ values from Eq. 25 rapidly approach the asymptotic values listed in Table 4. These values will be compared subsequently with values obtained from a parameter study.

θ :	γ
2	1.33
4	1.17
8	1.08

Table 4 Values of γ from Eq. 25 with $\theta = 1.5$ and $Z = \infty$

It must be recognized that the assumption utilized in developing the above relations between β and θ are not strictly true (i.e., the maximum elastic displacement of a system, S_d , does not exactly equal $\mu_{\Delta}\Delta_y$). In fact, it has been shown earlier that inelastic displacements will systematically exceed elastic ones for large values of Z , for structures with periods shorter than the predominant period of the ground motion, and for structures subjected to large geometric nonlinearities. Thus, one would expect systematic errors in the values of β predicted from θ in these cases. Equation 24 will likely underestimate the required value of β and Eq. 25 will unconservatively overestimate θ . The data presented so far would suggest that β values in excess of $1/a$ (design strength increases are bigger than strength losses due to geometric nonlinearities computed at Δ_e) would be required in some cases. This would correspond to negative values of θ . These errors will be examined subsequently.

Parameter Study to Identify Required γ Values. -- While Eqs. 20b and 25 indicate useful relationships, they do not indicate the values of γ (or θ) that are required to limit displacement ductilities under the effects of geometric nonlinearities to particular values. To do this the structures analyzed previously were re-analyzed for different strengths such that the computed ductilities equaled those for the system with geometric nonlinearities disregarded. A specially modified version of the NONSPEC computer program was utilized for this purpose. The search procedure employed identified the strongest structure needed to develop the required ductility (to within a specified tolerance on the required strength),

Figure 20 presents the γ ratios computed using the Taft record for structures with varying periods and differing α and Z values. Little systematic variation in γ is noted with period. There appears to be a slight decrease in γ with increasing period and a local minimum in γ near 2 secs. period for α equal 10, 4 and 8.

There are significant systematic variations in γ with Z and α (γ is proportional to Z and inversely proportional to α). It should be noted that the required strengthening exceeds the strength loss due to geometric nonlinearities computed at the maximum computed elastic displacement Δ_e when γ exceeds $1 + 1/\alpha$ (namely, 1.5, 1.25 and 1.125 for α equal 10, 4 and 8, respectively). This situation occurs for nearly all of the cases in Fig. 20, except where $Z = 2$. As indicated above, in Eq. 1 would have to be taken as a negative number for these cases to produce conservative results,

Table 5 lists the average values of γ required computed for various periods as a function of α and Z . The average required strength increases range from a low of 12 % for $Z = 2$ to 147 % for $Z = 8$.

Z	$\alpha = 8$			$\alpha = 4$			$\alpha = 2$		
	Comp. (Ave.)	Eq. 25 $\alpha=1.5$	Eq. 26a Eq. 26b	Comp. (Ave.)	Eq. 25 $\alpha=1.5$	Eq. 26a Eq. 26b	Comp. (Ave.)	Eq. 25 $\alpha=1.5$	Eq. 26a Eq. 26b
1	1.00	1.17	1.00 1.00	1.00	1.33	1.00 1.00	1.00	1.67	1.00 1.17
2	1.12	1.13	1.00 1.00	1.24	1.25	1.00 1.20	1.36	1.5	1.20 1.45
4	1.16	1.10	1.07 1.22	1.38	1.21	1.35 1.48	1.75	1.42	1.70 1.80
6	1.31	1.10	1.31 1.38	1.63	1.19	1.65 1.68	2.14	1.39	2.08 2.07
8	1.53	1.09	1.51 1.51	1.88	1.19	1.91 1.84	2.47	1.38	2.40 2.34

Table 5 Strength Ratio γ for Taft Earthquake

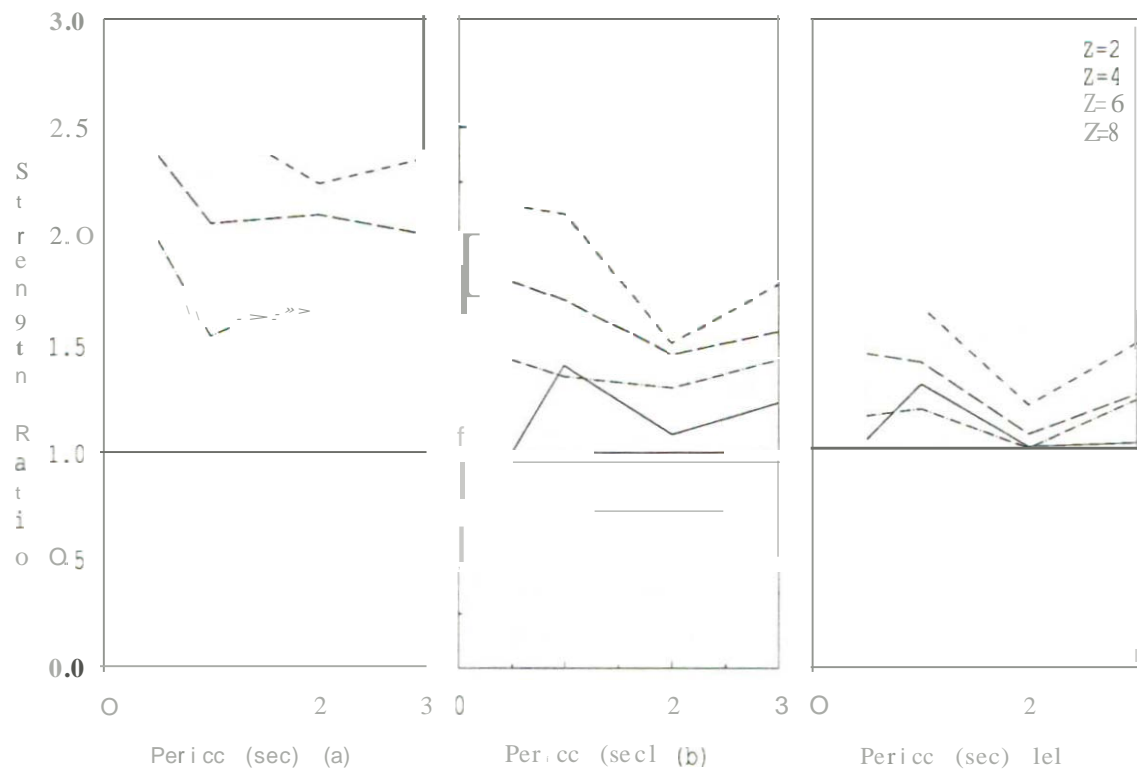


Fig. 20 Gamma. Values **EPI** models. Taft 569E. 1952 Kern County. a) $\alpha = 2$. b) $\alpha = -1$. c) $\alpha = S$.

The tabulated values of y , as anticipated from Eq. 25, depend systematically on Z and θ . The value of y is inversely proportional to θ . However, contrary to Eq. 25, the larger the Z value, the larger is the required increase in design strength, y . Thus, θ cannot be a constant, but must decrease (and in fact become negative) with increasing Z .

The computed values of y are usually much larger than predicted with Eq. 25 using $\theta = 1.5$. For example, for $Z = 4$ and θ equal to 8, 4 and 2, the *average* computed y values are 1.16, 1.38 and 1.75, respectively. These values are 1.05, 1.14 and 1.23, respectively, times those values predicted by Eq. 25. Other values of y may be found in Table 5.

Tabulated values in Table 5 based on Eq. 25 consider θ to equal 1.5. Values of y would decrease for larger values of θ (like 2 as used in the New Zealand Standard Code), generally decreasing the reliability of the simplified design values. The values of y in Table 5 for $Z = 8$ corresponding to Eq. 25 are close to the asymptotic values listed in Table 4.

Regression analysis of the data obtained from these analyses for the Taft record result in the following *approximate* relation for y :

$$y = 1.07 \cdot Z^{112/\theta - 113} > 1.0 \quad (26a)$$

Values obtained from this relation are included in Table 5. Relatively good agreement is obtained with the y values computed from the parameter study, except for small values of Z and large θ values. Better correlation may be found with the following more complex relation:

$$y = 1.43 \cdot Z^{0.311/\theta - 0.284} > 1.0 \quad (26b)$$

Both relations produce similar results, as indicated in Table 5. Results for Eqs. 26a and 26b are plotted in Fig. 21. The determination coefficient (R^2) for the latter regression analysis is 0.92, indicating very good fit to the data, even though the form of the equation does not represent any specific physical consideration. The result of the regression analyses has the highest error (12%) for low values of Z where the required increase in strength is the least.

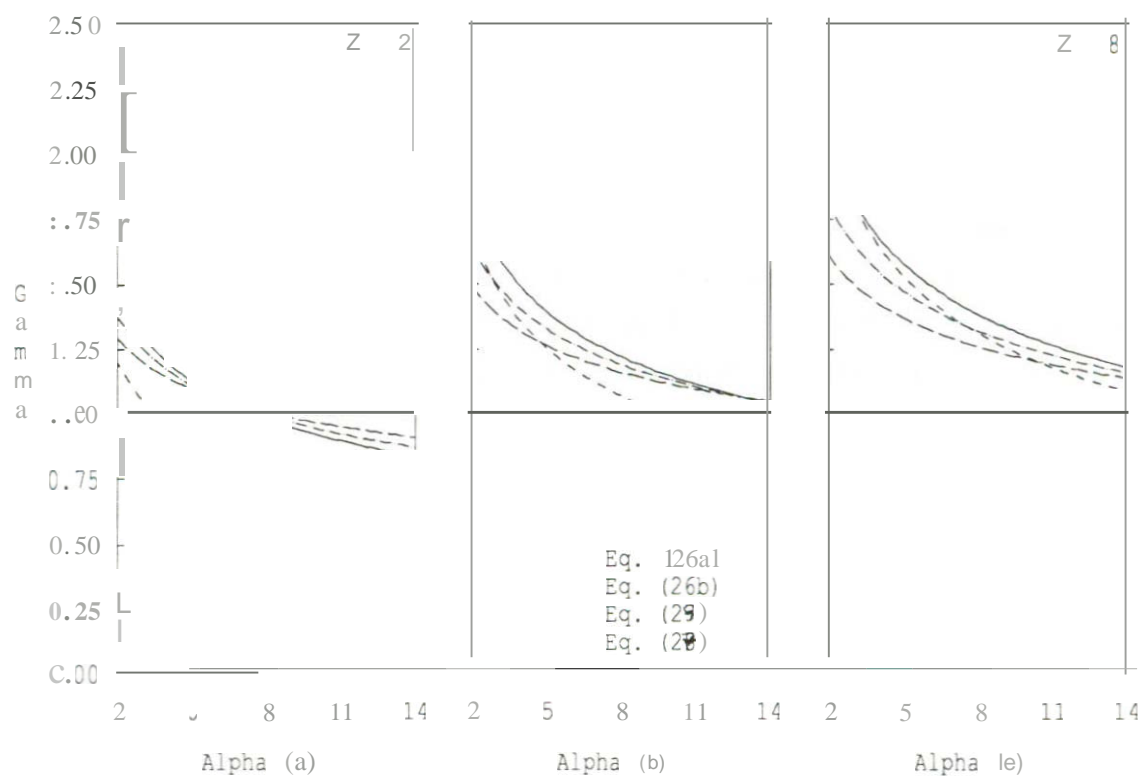


Fig. 21 Regression Analysis.

On the other hand, for Eq. 25 with $\epsilon = 1.5$, Z values would have to be limited to no more than 4.2 and (slightly more than) 2 for the predicted increase in strength to be within 10% of the required value for α values of 8.4 and 2, respectively. More restrictive limits would be needed for $\epsilon = 2$.

It should be noted that the inelastic response of many of the systems analyzed was very sensitive to changes in strength. For example, in several cases a reduction in strength by as little as 1% resulted in collapse instead of the stable response at the target ductility. Thus, it would be desirable to apply a factor of safety to either Eqs. 25, 26a or 26b.

Similar data was obtained using the Oakland Wharf record. However, it should be noted in Fig. 22 that the values of γ for several systems with $\alpha = 2$ are infinite. That is, the structures collapse regardless of the level of strengthening. This situation corresponds to cases where the target ductility demands for a system approach or exceed μ_c . Table 3 indicates that several of the elasto-perfectly plastic systems (with no $P-\Delta$ effects included) develop ductilities in excess of μ_c for α equal to 2 and periods of 0.5 or 1.0 secs. Simply strengthening such structures cannot, by definition, avoid collapse. This situation was particularly severe for this soft soil record, since many of the systems with periods of 0.5 and 1.0 secs. had ductility demands much larger than Z . Moreover, Takizawa and Jennings (1980) showed that the dynamic stability limit may be far smaller than μ_c . Given the large uncertainty in peak displacement demands for inelastic systems and the substantial increases in these ductility demand resulting from even moderate geometric nonlinearities, factors of safety (α) of 2 or 4 against static collapse may not be sufficient to reliably maintain seismic structural integrity.

In Fig. 22, it can be seen that for α equal to 2, it was possible only for the case of $Z = 2$ to fully achieve the design goal. It was possible to achieve the equal ductility criterion for only 75%, 75%, and 50% of the structures considered with Z equal to 4, 6 and 8, respectively,

The values of γ vary as before in being proportional to Z and $1/\alpha$. However, greater systematic variation with period is seen. Relatively small γ values are required for structures with a 2 sec period. In fact, the required values of γ are less than unity for most systems with a period of 2 secs. and α values taken to be equal to 4 or 8. This improved behavior appears to be a consequence of the predominant period of the ground motion

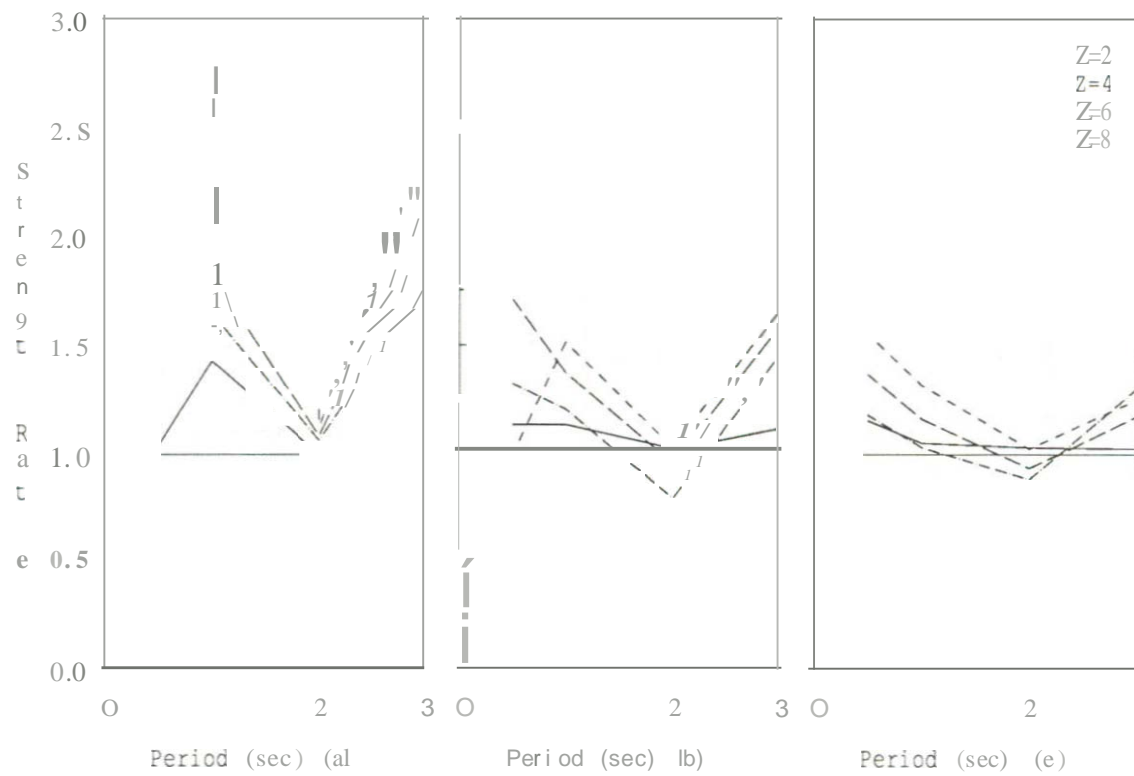


Fig. 22 Gamma. Values EPP models. Oakland Wharf 305 Deg, 1989 Loma Prieta. a) $\alpha = 2$. b) $\alpha = 4$. c) $\alpha = 8$.

being approximately 1.7 secs. The elongation of period due to geometric nonlinearities and yielding would shift the structure away from this peak in the elastic spectrum. This beneficial effect of yielding for structures at or slightly beyond the predominant period of the ground motion has been noted by others. Discounting the γ values at 2 sec period one does not see a significant variation in γ with period.

Average values of γ needed for the Oakland Wharf record to match the ductility demands on systems with and without $P-\Delta$ effects are listed in Table 6. In computing these averages, cases where γ is either infinite or less than unity have been disregarded. It is not believed that one should take advantage of the cases where geometric nonlinearities appear to help the structure resist inelastic action. It is interesting to note that the *average* γ values in Table 6 are often smaller than those obtained in for the Taft record.

Z	$\alpha = 8$			$\alpha = 4$			$\alpha = 2$		
	Comp. (Ave.)	Eg. 25 8=1.5	Eg. 27	Comp. (Ave.)	Eg. 25 8=1.5	Eg. 27	Comp. (Ave.)	Eg. 25 8=1.5	Eg. 27
I	1.00	1.17	1.00	1.00	1.33	1.00	1.00	1.67	1.12
2	1.06	1.13	1.01	1.10	1.25	1.14	1.22	1.5	1.29
4	1.17	1.10	1.16	1.32	1.21	1.31	1.49	1.42	1.49
6	1.23	1.10	1.26	1.55	1.19	1.43	1.66	1.39	1.62
8	1.28	1.09	1.34	1.39	1.19	1.51	1.77	1.38	1.71

Table 6 Strength Ratio γ for Oakland Wharf Record

It should also be recognized that the ductility demands for the Oakland Wharf record are significantly higher than the value of Z in many cases, and often much higher than the ductility demands computed for the Taft record. In many cases, the ductility demands required by the Oakland Wharf record are unacceptably large. If the ductility demands on the systems with $P-\Delta$ effects included were to be limited to a more reasonable

level, say 1.25 or 1.5 times the value of Z , or to a particular drift index, the required values of γ would be substantially larger. Additional nonlinear analyses are needed to identify these cases, however.

A regression analysis of the data in Table 6 results in the following relation for γ :

$$\gamma = 1.27 \cdot Z^{0.204} / \alpha^{0.18} > 1.0 \quad (27)$$

This relation is similar to Eq. 26 and is also plotted in Fig. 21. The results of the two equations are surprisingly similar given the vast differences in the characteristics of the two ground motions. This suggests that a more meaningful relation for γ may be obtained by extending the parameter study to a wider range of ground motions. Time constraints do not allow this at present. However, for the two records considered herein the following average relation (see Fig. 21) is obtained:

$$\gamma = 1.35 \cdot Z^{0.253} / \alpha^{0.233} > 1.0 \quad (28)$$

It may be that a more detailed study would identify a general relation of the form:

$$\gamma = a \cdot (Z^b / \alpha^c) (T_n / T_g)^d T_d^e > 1.0 \quad (29)$$

in which a, b, c, d and e are empirical parameters obtained from the regression analysis of the data;
 T_d is the duration of ground motion;
 T_g is the predominant period of the ground motion at the site.; and
 T_n is the natural period of the structure,

This general form would account for the observed sensitivity of response to soil conditions and to duration of shaking. A slightly better (e.g., logarithmic, $Z \cdot (a + O/(Za - O, \text{ etc.}))$ formulation may result in improved results for low values of Z and for site effects, This appears to be a promising approach to this problem.

In view of the cases where strengthening cannot achieve the equal ductility goal (or even stability) caution should be used in employing the above statistical relations, especially for low α and period, or high Z . More analyses are needed to establish acceptable factors

of safety these and other related equations. The required factor of safety may be a function of period, α and Z.

Comparison with Other Empirical Relations. -- Bernal (1987) suggests a strength amplification relation of the form:

$$\gamma = (1 + \lambda p) / (1 - p) \quad (30)$$

in which λ is an empirical factor obtained from a parameter study in which a predetermined ductility demand is achieved through numerical iteration. While Bernal detected a variation of γ with period (slightly larger γ values for short period structures) this variation was not sufficiently significant or regular for inclusion in his empirical relation. For the bilinear hysteretic systems studied by Bernal,

$$\lambda = 1.87P(\mu_{\Delta} - 1) \quad (31)$$

in which μ_{Δ} = the actual ductility developed by the system: and
 $P = 1$ for the mean value or 1.44 to represent the mean plus one standard deviation.

If it is assumed that $\mu \approx Z$ and Eqs. 14 and 31 are substituted into Eq. 3D, the strength amplifications required by Eq. 30 are only 19%, 47% and 25% larger than those estimated using Eq. 28 for $Z = 4$ and α values of 2, 4 and 8, respectively. This agreement further supports the idea that a standard amplification equation may be established. However, in the method used by Bernal one must know the actual ductility demand on the structure, which as shown in Fig. 13 may differ significantly from the target value. Thus, the amplification factor should be more appropriately based on the methodology used in design (i.e., the ratio of fully elastic demand to the plastic strength of the structure, 2).

Rosenblueth (1965) also developed an amplification factor on the basis of first principles. It is given by the relation:

$$\gamma = 1 / (1 - \mu_{\Delta} p) \quad (32)$$

For the assumptions introduced above this relation results in strength amplifications 23% greater, and 4 % and 3 % less than obtained with Eq. 28 for $2 = 4$ and α values of 2, 4 and 8, respectively. Equation 32 has been incorporated in the Mexican Seismic Code (1977) for buildings.

Another amplification relation was suggested in the tentative seismic provisions suggested by NEHRP (1988) for buildings. This is of the form:

$$y = 1/(1 - p) \quad (33)$$

which is similar to Eq. 32 except for absence of the ductility term. Bernal (1987) has shown for single degree of freedom systems with bilinear hysteretic restoring force characteristics that Eq. 33 is very unconservative even for small ductility demands.

Effect of Earthquake Duration. As indicated previously, a major parameter influencing the inelastic response of systems considering geometric nonlinearities is the duration of motion. To assess this effect simply, two copies of the Taft record were run back to back, simply doubling the duration of the ground motion. The structural model considered is the same elasto-plastic model used in the previous analyses.

The results are summarized in Tables 7 and 8. In the cases considered here, a large ductility, but a small $P-\Delta$ effect, was considered; i.e., 2 was taken to be 8 and α also equalled 8. The augmented strengths (2^* -- corresponding to R^*y) were obtained as before to achieve the same ductility for structures with and without $P-\Delta$ effects. For these cases, the ductility demands for the Taft record range from 8 to 13, depending on period. When the duration of the record is doubled by adding the second Taft record (column 4 in Table 7) the structure collapses for a period of 0.5 secs, and ductility demands increase by factors ranging from 250% to 520 % for other periods.

These results, along with those of earlier studies (Ziegler (1980), Mahin (1976), Jennings (1968)), clearly illustrate the important influence of duration on the inelastic response of structures with even small geometric nonlinearities. They also suggest that the cases identified before where Eq. 1 may be unconservative may be even more unconservative if longer duration motions, associated with larger magnitude earthquakes, are anticipated. Cases with low 2 and α , which could be treated satisfactorily with Eq. 1,

may also be vulnerable to the adverse effects of geometric nonlinearities, if the duration of motion is increased. The influence of duration on inelastic response needs to be better quantified.

To assess possible effects of directional biases within the Taft record itself, the sign of the Taft record added to the end of the first one is reversed. The results (column 6 in Table 7) indicate decreases in the overall ductility demands. However, the ductility demands are still unacceptably large, except for the most flexible of the systems. The reversed directionality of the second Taft record tries to overcome the **P-Δ** effects and drive the structure back towards the origin. The directional bias due to the **P-Δ** effects is so great, however, that this is not possible, even for cases where the second Taft record is doubled in amplitude (column 7 in Table 7).

Period (sec)	z^* 2 = 8	Earthquake Case				
		Taft	Tan + Taft	Difference (4)-(3)/(3)	Tatt-Tatt	Taft -2(Taft)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.5	4.73	13.3	oo	oo	56.6	25.6
1.0	4.76	12.1	62.9	5.20	34.5	17.0
2.0	6.58	8.7	23.3	2.68	14.9	7.9
3.0	5.34	9.5	24.0	2.53	11.1	7.2

Table 7 Effect of Earthquake Duration on Displacement Ductility Demands for Structures with Geometric Nonlinearities Included ($z = 8$ and $\alpha = 8$)

This situation is much different for elasto-plastic systems which do not have **P-Δ** effects included. Results similar to Table 7 are shown in Table 8 for elasto-perfectly plastic systems. It can be noted that there is little increase in displacement ductility due to the doubling of the earthquake duration. The main effect here is that the residual displacements present at the end of the first record are retained as the starting point for the second record. On the same basis, the ductility demands do not change very much at all when the second earthquake is reversed in sign (directionality effects cancel; small differences are due to the dynamic conditions present at the end of the first record).

Period (sec)	Z Z= 8	Earthquake Case			
		Taft	Taft + Taft	Difference (4)-(3)/(3)	Taft-Taft
(1)	(2)	(3)	(4)	(5)	(6)
0.5	4.73	6.5	6.7	1.05	6.5
1.0	4.76	7.8	10.0	1.28	7.8
2.0	6.58	6.8	6.8	1.00	6.8
3.0	5.34	6.8	10.3	1.51	6.8

Table 8 Effect of Earthquake Duration on Displacement Ductility Demands for Structures with Geometric Nonlinearities Ignored (Z = 8).

It should be recognized that the effects of shorter duration events, from smaller magnitude earthquakes (like the Parkfield record), would not be as severe even though the peak acceleration may be large (e.g., Takizawa and Jennings (1980)). The influence of duration needs to be carefully considered in evaluating geometric nonlinearities.

Stiffness Degrading Systems, -- Stiffness degrading systems, like ductile reinforced concrete structures, have been observed to have about the same maximum displacements and smaller residual offsets than those corresponding elasto-perfectly plastic systems. A series of simple analyses are presented herein to assess this situation for the case where geometric nonlinearities are present. The few studies performed in the past suggest that the effect of geometric nonlinearities may not be as severe for these systems as for elasto-plastic systems. However, many of these models (e.g., Ziegler (1980)) do not properly account for **P-Δ** effects on the re-loading branch of the hysteretic curves. The NONSPEC program was specially modified for these analyses to correct for the deficiencies of these earlier studies.

The structural idealization considered herein is the same as that idealized previously, except the shape of the hysteretic loop is of the form shown in Fig. 7. This hysteretic model is a stiffness degrading model commonly used in the analysis of reinforced concrete structures. However, the loops have been tilted to account for **P-Δ** effects. It has been mentioned previously that **P-Δ** effects may not be so severe for stiffness degrading models, because the apparent stiffness and strength on branches A-B and C-D in Fig. 7 are less than for equivalent elasto-plastic models. Consequently, it is easier for the response to overcome **P-Δ** induced forces and return towards the origin. However, as can be seen in

Fig. 7, there still may be considerable resistance to this restitution, because of the **P-Δ** effects present at points A and E, especially at large ductility values. This shift had not been incorporated in the earlier studies.

To illustrate the influence of geometric nonlinearities on seismic response consider Fig. 23. The structure analyzed is the same as that used in Fig. 14; i.e., a period of 2.0 secs. is considered, Z is set to 4 and 5% viscous damping is used for the Taft earthquake record. Figure 23a shows the response of a stiffness degrading model for which **P-Δ** effects have been ignored. The maximum ductility demand in this case is 3.67, identical to that developed by the elasto-perfectly plastic system. This illustrates the previous comment regarding the equality of maximum displacements for equivalent stiffness degrading and elasto-perfectly plastic systems. Geometric nonlinearities are introduced in the system corresponding to an α factor of 2. The **P-Δ** induced forces are superimposed on Fig. 23a, but the forces are not considered in the analysis presented in that figure.

When the **P-Δ** effects are considered, without any increase in strength, the stiffness degrading structure collapses, as shown in Fig. 23b, as did the elasto-plastic system (see Fig. 14b). The increase of strength needed to have the stiffness degrading system develop a ductility of 3.67 corresponds to a γ of 1.26. This value is 72 % of the value obtained for the bilinear hysteretic system. The response of the strengthened system is shown in Fig. 23c.

The overall results for the various combinations of structural parameters considered in this study are summarized in Table 9. It can be noted as before that the ductility demands for stiffness degrading and bilinear systems without geometric nonlinearities considered are similar (for comparison see Table 2). Systems without geometric nonlinearities are represented in the table by $\alpha = \infty$.

When geometric nonlinearities are introduced, the situation changes substantially. Increases in ductility demands due to **P-Δ** effects are generally far less for the stiffness degrading systems, especially for systems with low ductility demands (low Z factors) and high periods. None the less, stiffness degrading systems should have **P-Δ** effects considered in many cases. For example, 6 %, 63 % and 94 % of the structures considered in Table 9 would suffer a 20 % or greater increase in ductility demand when geometric nonlinearities corresponded to α equal to 8, 4 and 2, respectively. Greatest increases are

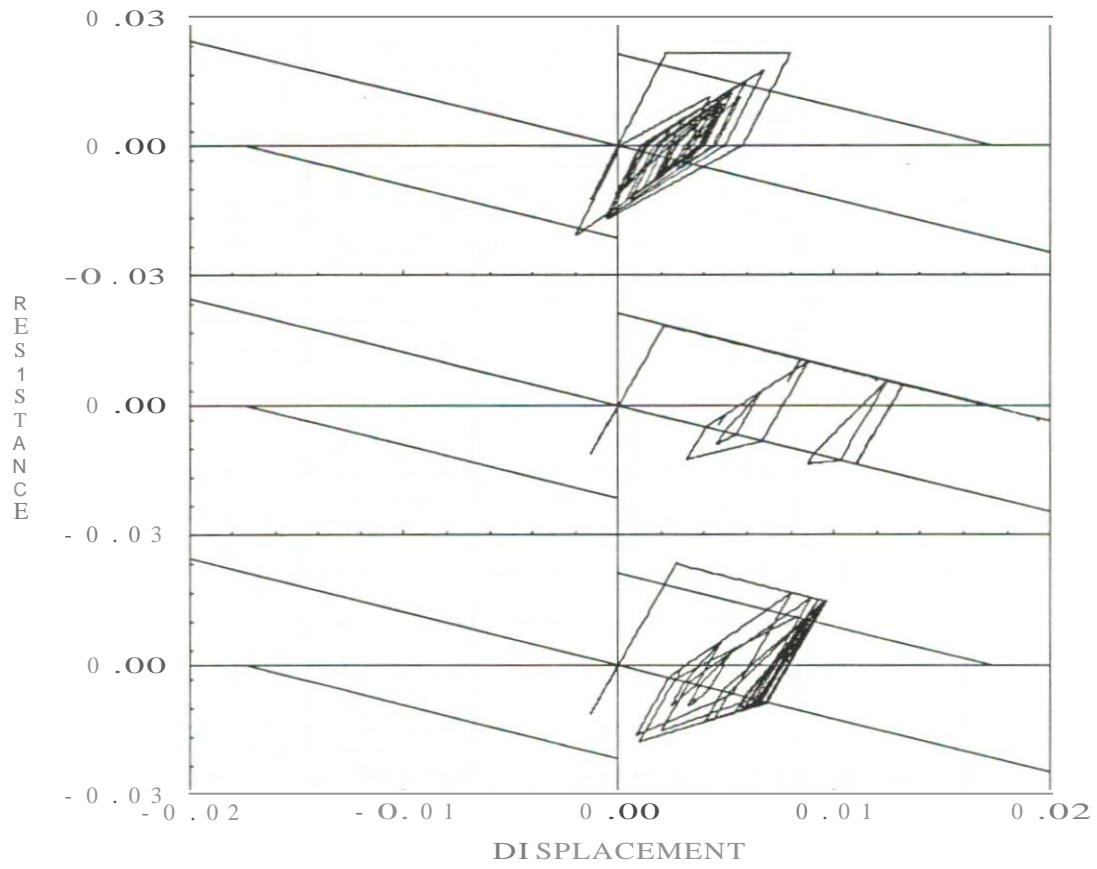


Fig. 23 Stiffness Degrading Models Hysteresis Loops. Taft S69E, 1952 Kern County, Period 2 sec, $Z = -1$, a) No P- Δ . b) $0=2$. $i = 1$. c) $0=2$. $i = 1.26$.

Z	Period or Collapse Ductility, μ_c		α			
			2	4	8	oo
2	μ_c		4	8	16	oo
	Period	0.5	ce	1.94 (13)	1.45 (-16)	1.72
		1.0	ce	2.20 (-3)	2.27 (0)	2.28
		2.0	1.77 (11)	1.68 (6)	1.64 (3)	1.59
		3.0	2.61 (72)	1.74 (14)	1.67 (10)	1.52
4	μ_c		8	16	32	ce
	Period	0.5	ec	4.87 (30)	4.09 (9)	3.74
		1.0	ce	ee	6.00 (12)	5.35
		2.0	ee	3.78 (3)	3.72 (1)	3.67
		3.0	ee	3.80 (13)	3.52 (4)	3.37
6	μ_c		12	24	48	ec
	Period	0.5	ce	ce	7.74 (2)	7.59
		1.0	oo	oo	8.90 (10)	8.12
		2.0	oo	7.48 (26)	6.33 (6)	5.95
		3.0	oo	6.95 (20)	5.95 (2)	5.81
8	μ_c		16	32	64	oo
	Period	0.5	oo	17.61 (39)	12.06 (-5)	12.65
		1.0	oo	oo	13.88 (28)	10.88
		2.0	oo	15.18 (75)	9.08 (5)	8.67
		3.0	oo	11.58 (43)	8.09 (2)	7.92

Table 9 Displacement Ductility Demands for Taft Record; Stiffness Degrading Systems ;
Values in Brackets Indicate Percentage Change from Case with $\alpha = ce$,

observed for smaller α and higher Z values. Special attention to geometric nonlinearities would have to be paid in these cases.

From Table 2 it can be seen that 81 %, 88 % and 100 % of the bilinear hysteretic structures have ductility demand increases greater than 20 % when α equals 8, 4 and 2, respectively. It is thus clear that reinforced concrete structures are not as sensitive to $P-\Delta$ effects as systems exhibiting bilinear restoring force characteristics. None the less, $P-\Delta$ effects can be very important for either type of system when $Z \geq 4$ or $\alpha \leq 4$.

The required strength increases (γ) needed for equal ductility demands with and without $P-\Delta$ effects are plotted in Fig. 24 for the stiffness degrading systems. The same basic variation of γ with α and Z are noted by comparing results in Figs. 20 and 24. However, it is clear that the γ values are typically less for the stiffness degrading systems than for the bilinear ones. In addition, the value of γ for the stiffness degrading structures decreases erratically with increasing period as was seen for the bilinear structures, but the trends for the two types of systems differ in detail. These differences may be an attribute of the Taft earthquake record, and additional ground motions need to be considered in order to better delineate differences in γ associated with period. In some cases, values of γ are less than one, indicating that the response is helped by the addition of geometric nonlinearities. As before, this is associated most probably with a shift in the period of the structure away from a local peak in the response spectrum due to the $P-\Delta$ effects and yielding.

Average values of γ obtained (for various periods) are listed in Table 10, as a function of α and Z . Values of γ less than unity have again been ignored in computing the averages. The average values of strengthening needed are smaller than obtained for the elasto-plastic systems (compare Tables 5 and 10).

The ratios of γ values for the stiffness degrading systems to those for comparable bilinear systems are plotted in Fig. 25. The ratios vary in an irregular manner with Z and period. However, the γ values for the stiffness degrading systems are generally between 60 and 100 % of the values for bilinear systems. Smaller ratios tend to develop for systems with larger Z . This may be because the differences in the shapes of the hysteretic loops do not become pronounced until large ductilities develop (i.e., for large Z values).

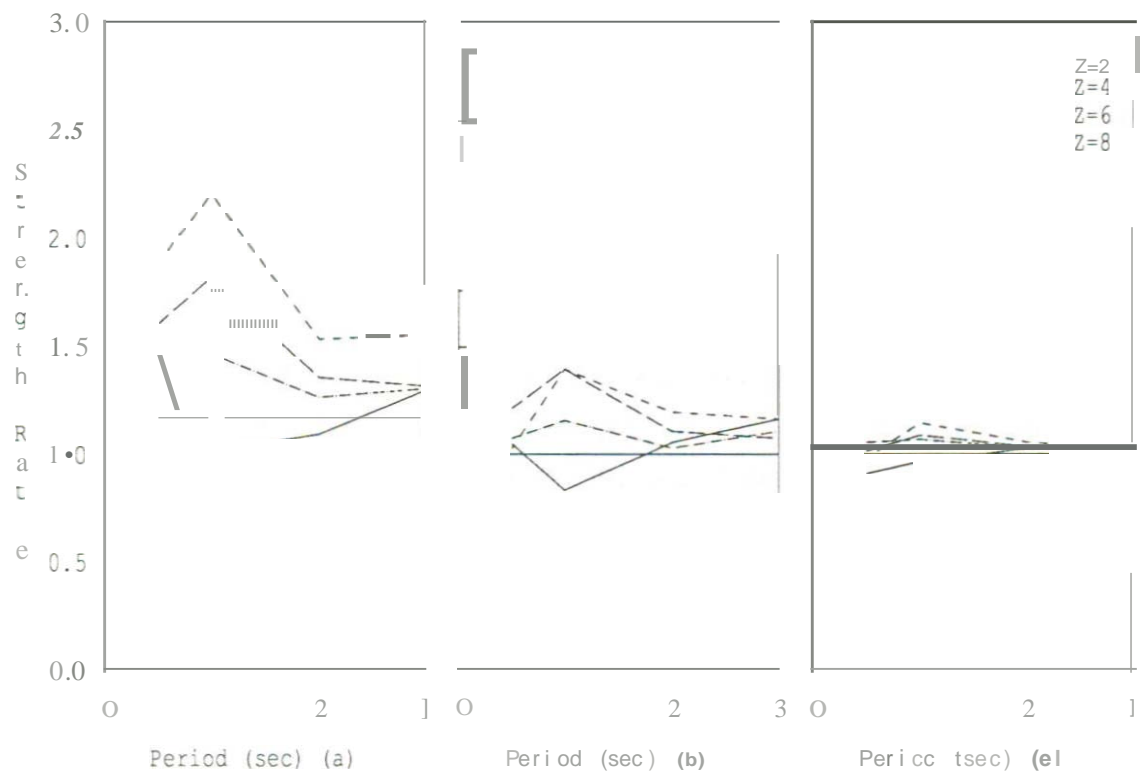


Fig. 24 Gamma. Values 5D models. Taft 569E. 1952 Kern County. a) $q = 2$. b) $q = 4$. c) $q = 8$.

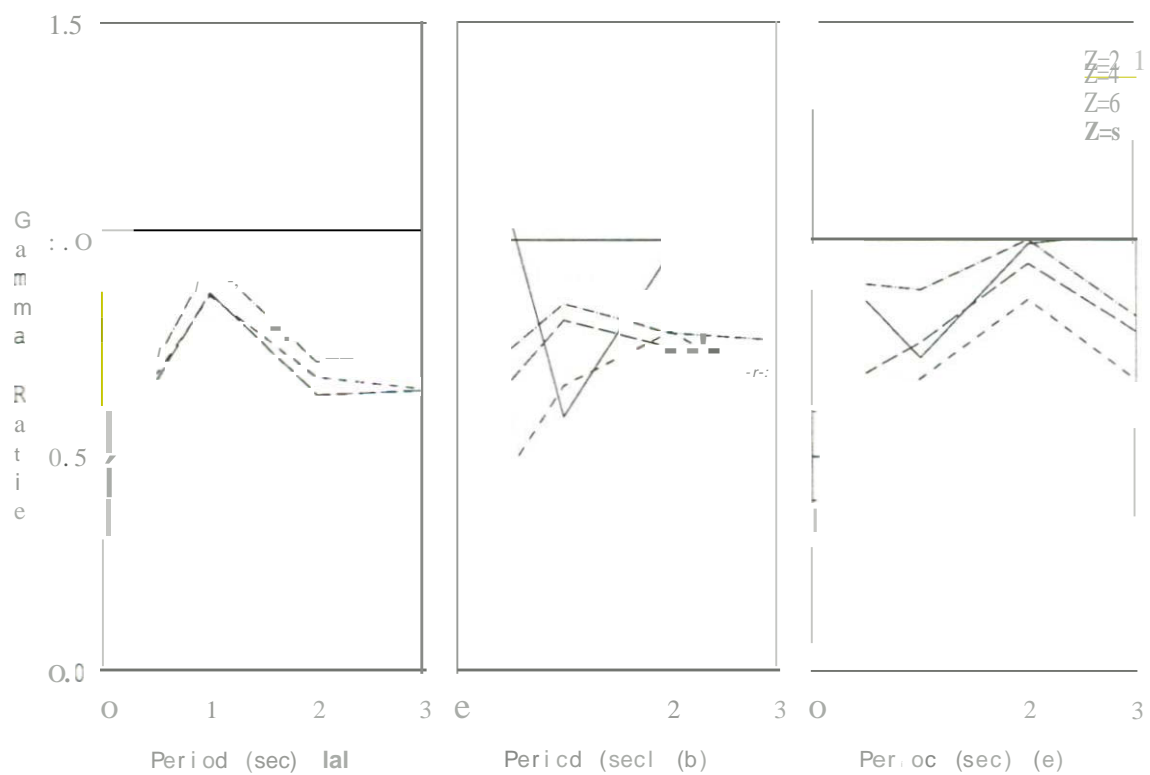


Fig. 25 Gamma Ratio for Stiffness Degrading and Elasto Perfectly Plastic Models. Taft S69E, 1952 Kern County. a) $a = 2$. b) $a = 4$. e) $a = 8$.

Z	$\alpha = 8$			$\alpha = 4$			$\alpha = 2$		
	Comp. (Ave.)	Eq. 25 8 = 1.5	Eq. 34	Comp. (Ave.)	Eq. 25 8 = 1.5	Eq. 34	Comp. (Ave.)	Eq. 25 8 = 1.5	Eq. 34
1	1.00	1.17	1.00	1.00	1.33	1.01	1.00	1.67	1.18
2	1.05	1.13	1.00	1.08	1.25	1.09	1.20	1.5	1.29
4	1.04	1.10	1.01	1.09	1.21	1.19	1.36	1.42	1.40
6	1.04	1.10	1.06	1.19	1.19	1.25	1.52	1.39	1.47
8	1.07	1.09	1.10	1.19	1.19	1.29	1.79	1.38	1.52

Table 10 Value of γ for Stiffness Degrading Systems for the Taft Earthquake

For a Z value of 4, the ratios of γ values average around 80 % for α values of 2 and 4 and 90 % for $\alpha = 8$. It may be possible that the relation for bilinear hysteretic systems may prove suitable after investigation for stiffness degrading structures since it may provide a factor of safety roughly of the correct order of magnitude and that increases appropriately properly with $1/\alpha$ and Z .

Assuming required strength increases in excess of 10 % should be considered in design, $P-\Delta$ effects need to be considered for stiffness degrading systems for α equal to 8 for structures with periods around 1 sec and $Z = 8$. For α equal to 2, virtually all structures should have $P-\Delta$ effects explicitly considered. For α equal to 4, geometric nonlinearities should be considered for roughly half the cases considered. However, the variation in γ with period may be sensitive to the record selected; so additional work is needed to make conclusions as to which cases could have $P-\Delta$ effects safely ignored.

Table 10 also suggests that the previous general conclusions regarding the general unreliability of Eq. 1 with $8 = 1.5$ or greater still hold for stiffness degrading systems. However, the ability of Eq. 1 to compensate for the $P-\Delta$ induced forces is improved due to the lower sensitivity of stiffness degrading systems to these effects. Conservative results are obtained using $8 = 1.5$ for α equal to 4 and 8. However, the variation of required strengthening still varies incorrectly with Z resulting in large over-conservatisms at small Z

values. For α equal to 2 the predicted strengthening obtained using Eq. 1 with $\theta = 1.5$ results in unconservative results for $Z \geq 6$.

However, in most cases it is not necessary to consider the full elastic displacement of the stiffness degrading structure in order to compute the increased design forces needed to compensate for $P-\Delta$ effects. As indicated before, this situation corresponds to γ values of 1.5, 1.25 and 1.125 for α values of 2, 4 and 8, respectively. For the bilinear hysteretic systems subjected to the Taft earthquake, nearly all of the cases considered needed to have displacements larger than Δ_e used to compute the corrective forces (using Eq. 2). For the stiffness degrading systems, this case is only encountered for two cases with large Z and small α .

A regression analysis for γ has for the stiffness degrading systems results in the following relation:

$$\gamma = 1.392 - 0.12/\alpha + 0.233Z > 1.0 \quad (34)$$

The coefficient of determination for this relation is only 0.75 indicating poorer correlation than obtained for the bilinear systems. Additional efforts are required to refine this relation using other relational forms and adding additional ground motions. As indicated previously, the scatter in the results resulting from uncertainties in ground motion characteristics and structural parameters needs yet to be interpreted in order to establish appropriate factors of safety for such equations.

The values obtained from Eq. 34 are also listed in Table 10. For cases of interest (say, $\gamma \geq 1.10$), the relation works reasonably well for $\alpha = 4$ and 8, but is unconservative for large Z (≥ 6) when $\alpha = 2$. Additional ground motions need to be considered and other forms of relations need to be investigated to improve Eq. 34.

Comments. -- Results for idealized structures have shown that Eq. 1 estimates strength enhancements needed to compensate for $P-\Delta$ effects that vary improperly with Z . The value of θ in Eq. 1 should ideally vary with α and Z . For a constant value of θ , equal to or larger than 1.5 (as adopted for the Terminal Separator structures), strength enhancements suggested for bilinear hysteretic systems were overly conservative for small Z and substantially unconservative for large values of Z . For stiffness degrading systems,

the degree of unconservatism decreases to acceptable levels for all but those systems with large Z and small u , None the less, the improper form of the equation results in overly conservative strength requirements when ductility demands are small or moderate.

Several relations were developed on the basis of regression analyses of the results of a limited parameter study. These relations proved much more reliable than either Eqs. 1 or 2. Additional effort is needed to assess how these relations might change in view of the uncertainties in ground motion or structural characteristics, and to determine methods for applying these results to practice.

DESIGN IMPLICATIONS

The analyses presented in the previous section provide considerable insight into the effects of geometric nonlinearities on inelastic response. Based on these analyses, it appears that Eq. 1 does not provide a desirable degree of reliability. Equation 2 also has limitations in that it is difficult to estimate the maximum inelastic deformation of the structure. It was shown that the elastic displacement of the structure does not conservatively estimate the inelastic displacement for bilinear hysteretic systems and may give unconservative approximations for many important stiffness degrading systems.

Empirical relations like those presented in Eqs. 28 and 34 make it possible to estimate strength increases needed assure that the ductility demand on a system with **P-Δ** effects considered is no more severe than if these effects were disregarded. Several issues need to be addressed before attempting to apply these results to actual structures. In this section, some of these issues will be briefly addressed and a basic design methodology will be outlined. It is attempted to formulate a methodology compatible with current procedures. Before such a methodology can be implemented with confidence additional effort will be needed to improve the various relations, to refine the various steps, and to assess its accuracy and practicability.

Basic Issues. -- In order for the ideas introduced in last section to be applied, two parameters must be identified for the system, α and Z . These should be based on effective properties for an ideal elasto-plastic envelop curve developed for the structure which may exhibit deformation hardening as shown in Fig. 8.

The value of Z is not equal to the design value of Z used to reduce the elastic spectrum to obtain ultimate design moments in the columns, but rather it must represent the ratio of the moments in the member if the structure were to remain elastic (M_e) to the actual plastic moment capacity of the column (M_p') as designed. That is,

$$Z = M_e / M_p' \quad (35)$$

Thus, the column must be first designed for the applicable load conditions, and a specific size and reinforcement arrangement must be selected. Realistic material properties should be considered in computing the plastic moment capacity of the plastic hinge. This process is similar to that already used by Caltrans in the seismic design of frames when computing

plastic shears in columns and internal forces in bent caps. Thus, it should not introduce major complexities into the design process.

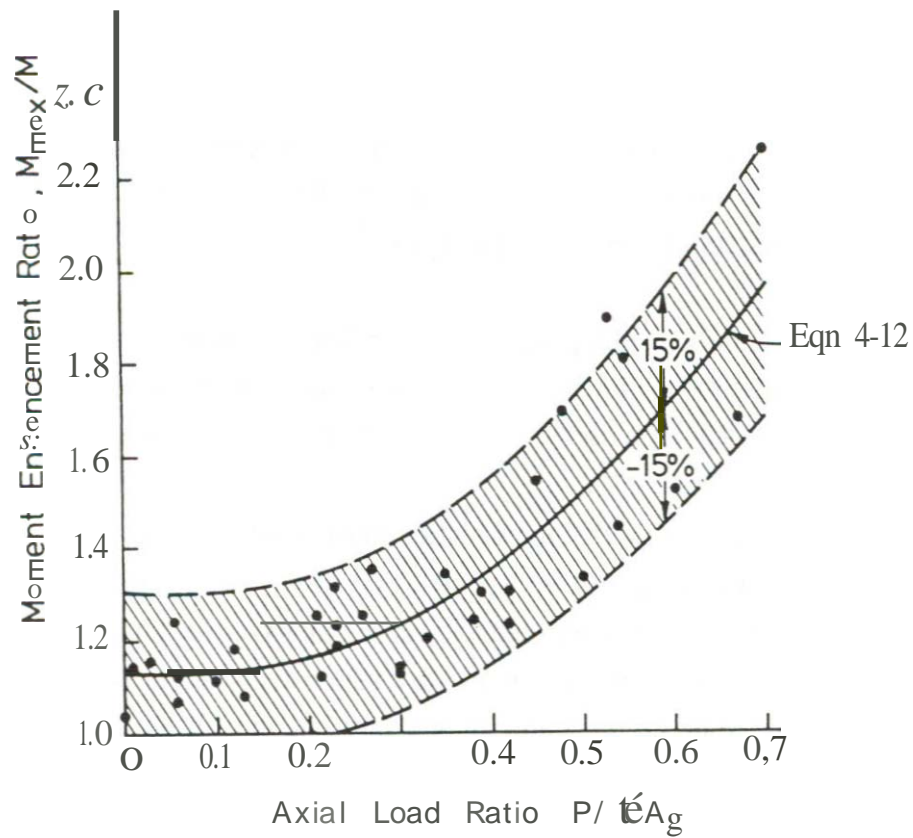
However, in the case of geometric nonlinearities, we are looking not for the maximum strength that can ever be developed by the member, but for the dependable plastic strength that can be counted on to resist the **P-Δ** effects. Thus, rather than estimating M_p by multiplying the nominal ultimate moment by a factor like 1.3, a smaller factor like 1.1 would be more appropriate. Figure 26 (Priestley and Park, 1984) shows the likely scatter in the plastic moment capacities of spirally reinforced columns as a function of axial load. For the range of axial loads commonly used in design practice a factor of 1.3 represents a reasonable upper bound on the probable plastic moment capacities. These large values may not always develop, and thus cannot be counted on to resist **P-Δ** effects. Consequently, the dependable plastic moment capacity could be estimated with a factor ranging between 1 and 1.2 times the nominal ultimate moment capacity.

The value of α is obtained by estimating the **P-Δ** forces (i.e., $P\Delta_e/L$) acting on a column at the full elastic displacement of the structure, Δ_e . This displacement value is easily obtained from the standard analyses carried out as part of the standard design process. It does not require computation of the shift in period of the structure due to **P-Δ** effects nor does it consider the increased elastic displacement in the structure due to the **P-Δ** induced forces. Since the structure will be stronger than the design moment (M_e/Z) due to the choice of section and reinforcement, the actual material properties used, strain hardening and other factors, the value of α should be based on the dependable plastic moment capacity of the plastic hinge. Consistent with the definition of α we would obtain the following approximate relation for a structure idealized as being elasto-perfectly plastic:

$$\alpha = R_y/(P\Delta_e/L) = M_p'/(P\Delta_e) \quad (36)$$

However, the structure will gradually strengthen from M_y to M_n to M_p , giving the system a deformation hardening that will counter any **P-Δ** effects. This effect can be estimated by idealizing the actual load - displacement relation for the column as a bilinear one with appropriate deformation hardening. It may be possible to conservatively estimate this effect by assuming a linear relation between the conditions at M_n and M_p' . Thus, one obtains:

$$\alpha = - M_n/[M_p' - P\Delta_e - M_n] \quad (37)$$



$$P/f_c' A_g < 0.1 \quad M_{max}/M_n = 1.13$$

$$P/f_c' A_g \geq 0.1 \quad M_{max}/M_n = 1.13 + 2.35 (P/f_c' A_g - 0.1)^2$$

Equation 4-12

Fig. 26 Ratio of Maximum Flexural Strength to Predicted Nominal Strength (M_n) as a Function of Axial Load Intensity (Priestley and Park, 1984)

Negative values of α indicate that the structure has a positive post-yield tangent stiffness even with **P-Δ** effects imposed. In such circumstances, geometric nonlinearities need not be considered.

The accuracy of these relations remains to be evaluated. For example, if small ductilities are actually developed in a column relative to its ductility capacity, M_p' will not fully develop, reducing Z and generally increasing α .

With α and Z identified it is possible to use relations like those developed in the previous section to determine the increase in column moment capacity needed to resist **P-Δ** effects. The relations may also be used to identify cases where **P-Δ** effects can be ignored.

In all of this, it is assumed that the structure to be considered can be idealized as a single degree of freedom system. It is expected that frames with multiple columns of similar height from relatively straight viaducts can be analyzed in this manner. Issues related to the ductility demands arising in multiple level viaducts and in systems subject to significant in-plane torsional motions are not addressed herein. It will be assumed that plastic hinges form in the columns and that the columns are pinned at one end.

For multiple column frames, it is possible to interpret the basic design method in terms of overall frame characteristics, rather than treating each column individually. Thus, the global **P-Δ** effect will be $\Sigma P \Delta_e / L$ and α can be based on:

$$\alpha = - \Sigma(M_n/L) / [\Sigma(M_p'/L) - \Sigma(P/L)\Delta_e - \Sigma(M_n/L)] \quad (38)$$

and

$$Z = \Sigma(M_e/L) / \Sigma(M_n/L) \quad (39)$$

where the summations are taken over all of the columns in the frame for the direction of motion in question.

Design Methodology, -- To see how these concepts would relate to the overall design procedures currently used by Caltrans, a brief outline of a methodology incorporating the checks for **P-Δ** effects is presented below. Only those steps directly effected by the **P-Δ** computations are presented,

1. Dynamic analysis of the structure should be performed based on cracked section properties and an appropriately selected smoothed response spectrum. In the absence of more definitive data, cracked section moments of inertia may be approximated as the gross section properties divided by 2.
2. Initial ultimate strength requirements are estimated by dividing the computed elastic member flexural demands M_e by Z . The value of Z is selected according to the *Bridge Design Specification Manual* or other project specific design criteria. It may be desirable to resize members to limit drifts to a specific upper bound. At the moment this value is not known, but drift indices on the order of 1 % to 2 % at the full elastic moment (M_e) condition are consistent with design practices used for other important structures.
3. Axial loads in the elements at collapse are estimated considering equilibrium, the locations and plastic moment capacities of plastic hinges, and the intensity of gravity loads likely to be present at the time of an earthquake. This step is the same as in current practice.
4. Columns should be sized based on the ultimate design moments (Step 2) and the axial loads determined in Step 3. The ultimate and plastic moment capacities of the columns should then be realistically estimated. This may be done by computing moment - curvature relations for the critical plastic hinge sections for appropriately selected axial loads. Alternatively, the dependable plastic moment capacity M_p' might be estimated as a factor (like 1.1) times the nominal ultimate moment capacity, M_n . The dependable plastic moment will differ from the maximum plastic capacity used to design the shear reinforcement and to determine internal forces in the remainder of the frame.
5. As with current design procedures it may be necessary to revise the design axial load estimates and the plastic capacities based on the results of Step 4.
6. Estimate Z and a using a lateral load - lateral displacement relation developed for the structure. Alternatively, it may be possible to use Eqs. 35 and 37; i.e.,

$$Z = M_e/M_n \quad (35)$$

and

$$\alpha = -M_n / [M_p' - P\Delta_e - M_n] \quad (37)$$

If the resulting α is negative, **P-Δ** effects need not be considered.

7. Use relations like Eqs. 28 or 34 (with an appropriately applied factor of safety) to determine the required strength $M_n^* = \gamma M_n$ to compensate for **P-Δ** effects. If γ is less than some tolerance level (like 1.1) **P-Δ** effects can be ignored. As noted previously, it appears that Eq. 28 may be conservatively used for stiffness degrading systems. Given the potential instability of structures with large γ (large Z , small α or periods less than the predominant period in the design spectrum), it may be prudent to apply an additional large factor of safety in these cases, or require nonlinear analysis to verify the design.
8. The column is redesigned if required for the new nominal capacity. This may require modification of the axial loads considered and an iteration of steps 4 through 7.
9. The maximum displacement of the system may be checked at this point. The previous parameter study did not focus on this issue, but noted that the displacement of the strengthened structure will be larger than that of the original structure without the **P-Δ** effects applied even though the ductility demands are the same. This is because the yield displacements will be bigger by the factor M_n^*/M_n ($\approx \gamma$). For structures with a period greater than the predominant period of the ground motion and for Z values less than or equal to about 4, the inelastic displacements may be conservatively estimated by $\Delta_e M_n^*/M_n$. In other situations, methods suggested by Newmark (Newmark and Rosenblueth, 1971) may be used.
10. The maximum plastic capacity M_p of the revised plastic hinge is now computed. M_p is then used to design the transverse reinforcement in the column for shear and confinement and to compute internal ultimate forces for the design of adjacent footings and bent caps as done currently.

SUMMARY AND CONCLUDING REMARKS

Based on a review of available literature and of analytical results presented in this background report, the following observations are made regarding the effects of geometric nonlinearities on seismic response, current design procedures and items requiring additional research.

1. When the effective tangent stiffness of a system can become negative following yielding, the maximum inelastic displacements can be substantially larger than predicted elastically. The severity of the displacement increase is proportional to P/Δ , the "effective" value of β and the duration of the earthquake.
2. Inelastic demands are underestimated for systems with and without geometric nonlinearities when the period of the structure is near or less than the predominant period of the ground. This may require special considerations in design for short, relatively stiff structures on firm ground or for most structures located on soft soil sites.
3. Previous analytical studies of buildings in which column plastic hinging is allowed indicate that $P-\Delta$ effects can substantially increase overall lateral displacements and ductility demands. The amount of increase is sensitive to the duration of motion.
4. The imposition of a strong column - weak girder design philosophy along with drift limits has been found to be a practical and effective means of limiting the adverse effects of geometric nonlinearities for building structures. A considerable range of drift limits has, however, been incorporated in various codes. Considering deformations corresponding to application of the full elastic design spectrum ($\beta = 1$) code limits on drift indices range from 1 % to 4 %. NEHRP (BSSC, 1988) provisions for inverted pendulum structures limit drifts to 1 % for important structures and 1.5 % for others. Analytical studies of concrete frame structures suggest satisfactory behavior for drift limits ranging from 1.5 % to 2 %. Disproportionate increases in drift are associated even for strong column - weak girder designs when elastic drifts exceed 2 %. Significantly, analytical studies have focused on situations requiring ductility demands less than 4, short to moderate ground motion durations, and strong column - weak girder plastic behavior. As a result, the building structures had overstrengths and post-yield deformation

hardening ratios substantially larger than expected for bridge structures. While a definitive drift index remains to be established, values of 1.5 % or less (computed elastically for $Z = 1$) appear appropriate.

5. The design equation:

$$M_{\Delta} = [1 + \mu_{\Delta}] P \Delta_y / \theta \quad (1)$$

is based on static considerations. For $\mathbf{C} = 2$, based on conservation of energy, or the more conservative $\mathbf{C} = 1.5$, this equation is generally unable to limit ductility demands for bilinear hysteretic systems with geometric nonlinearities to values similar to those computed when geometric nonlinearities are disregarded. Equation 1 requires strength increase that vary improperly with Z . As a result Eq. 1 is conservative for small Z values and unconservative for large Z values. For $\mathbf{C} = 1.5$, Eq. 1 underestimates required strengths by no more than 10 % only if $Z \leq 4$ for $\alpha = 8$ and $Z \leq 2$ for α equal to 2 or 4. For larger Z and lower α values large increases in ductility demands might be expected. The value of \mathbf{C} should change with Z and α ; however, for bilinear hysteretic systems, the value of \mathbf{C} would have to be negative (not physically meaningful) in most instances to achieve comparable ductility demands for systems with and without $\mathbf{P}-\Delta$ effects.

6. Equation 1 is much more reliable for stiffness degrading systems. In this case, the equation underestimates the required strength for equal ductilities for only cases of low α and high Z . However, the improper trend of the required strengthening with Z is noted (Table 10) and substantial levels of conservatism occur for small Z values.
7. The required increase in strength needed for a structure to achieve the same ductility with $\mathbf{P}-\Delta$ effects considered or ignored was determined for a few round motions. For bilinear systems subject to the Taft earthquake record the increased strength could be estimated considering the expression:

$$\gamma = M_n^*/M_n = 1.07 \cdot Z^{1/2} f_a^{1/3} > 1.0 \quad (26a)$$

or more accurately by:

$$\gamma = 1.43 \cdot Z^{0.31} \alpha^{0.284} > 1.0 \quad (26b)$$

Considering a record obtained on a soft soil site, a similar relation was obtained

$$\gamma = 1.27 \cdot Z^{0.204} \alpha^{0.18} > 1.0 \quad (27)$$

These expressions follow the trends exhibited by the structures and produce errors generally less than 10% for all structures considered. A composite relation was also established for the two dissimilar motions considered for bilinear hysteretic behavior as follows:

$$\gamma = 1.35 \cdot Z^{0.253} \alpha^{0.233} > 1.0 \quad (28)$$

More detailed studies are needed to identify the effects of other ground motions, the predominant period of the ground and the duration of ground shaking. A relation of the following form might then be developed:

$$\gamma = a \cdot (Z^b / \alpha^c) (T_n / T_g)^d T_d^e > 1.0 \quad (29)$$

8. The parameter study also indicated the preferential behavior exhibit by stiffness degrading systems. None the less, stiffness degrading systems should have **P-Δ** effects considered in many cases. For example, 6 %, 63 % and 94 % of the structures considered in Table 9 would suffer a 20 % or greater increase in ductility demand when geometric nonlinearities corresponded to α equal to 8, 4 and 2, respectively. Typically, γ values for stiffness degrading systems are between 60 and 100 % of the values for bilinear systems. Smaller ratios tend to develop for systems with larger Z . For a Z value of 4, the ratios of γ values for stiffness degrading systems to those for bilinear hysteretic systems average around 80 % for α values of 2 and 4 and 90 % for $\alpha = 8$. For the Taft earthquake record, the following empirical relation can be used to determine required strength increases:

$$\gamma = 1.39 Z^{0.12} / \alpha^{0.233} > 1.0 \quad (34)$$

As before, additional analyses are required to estimate the effects of other ground motions, site conditions and earthquake durations, and to determine appropriate

factors of safety, It was noted that Eq. 28 for bilinear hysteretic systems appears to provide a factor of safety that varies appropriately in proportion to the severity of the **P-Δ** effect.

9. The duration of ground shaking was found to be a very important factor influencing inelastic response of systems with negative post-yield tangent stiffness. Short duration motions may not have an important effect on the inelastic response of ductile structures even though peak ground accelerations may be quite high. Long duration motions may conversely, have a profoundly adverse effect on displacement and ductility demands. For example, a series of structures with a large ductility demand, but a small **P-Δ** effect, were considered herein; i.e., Z was taken to be 8 and α also equalled 8. The strengths of these systems were augmented to achieve for the Taft record the same ductility demands for structures with and without **P-Δ** effects. By doubling the duration of the Taft record all of the short period structures analyzed collapsed and ductility demands for other periods increased by factors ranging from 250% to 520 %. The potentially long duration motion associated with soft soil conditions or large magnitude seismic events suggests that special precautions may have to be taken in these circumstances to limit **P-Δ** effects,
10. A design methodology consistent with current Caltrans design procedures was outlined. The methodology utilizes the standard elastic analysis of the structure to determine the initial design forces and displacement levels. Once a trial design of the structure is completed, the severity of the **P-Δ** effects at the maximum elastic displacement (for $Z = 1$) is characterized by the parameter a :

$$a = -M_n / [M_p' - P\Delta_e - M_n] \quad (37)$$

The fraction $1/a$ represents the proportion by which the effective strength of the structure has been reduced by geometric nonlinearities. The actual capacity of the columns to resist the increased forces induced by the geometric nonlinearities is characterized by the effective Z value:

$$Z = M_e / M_n \quad (35)$$

Empirical relations like Eqs. 29 are then used along with the computed values of α and Z to determine the factor γ by which the strength of the column must be augmented in order to limit ductility demands in the structure when $P-\Delta$ effects are considered to those that would have developed had geometric nonlinearities not existed. That is,

$$M_n^* = \gamma M_n \quad (40)$$

While additional details need to be devised and the methodology needs to be applied to actual examples, it is expected that it will prove to be more reliable than methods based on either Eqs. 1 or 2.

11. It was noted that the residual displacement ductilities ($\Delta_{\text{permanent}}/\Delta_y$) in structures with even small amounts of post-yield deformation softening are very nearly equal to the maximum displacement ductility developed by the system. The systems tend to displace in only one direction and may fail to return towards the initial undeformed configuration. Two ramifications of this behavior remain to be addressed:

- a. The continued operability of the structure may be adversely affected by these permanent offsets, and
- b. The capacity of the structure is diminished by the $P-\Delta$ forces present in the offset configuration. As a result the ability of the structure to withstand the effects of aftershocks immediately occurring after a major earthquake needs to be assessed (alternatively, emergency shoring procedures may need to be devised prior to an earthquake for structures designed using large Z values and moderate to small a).

The focus of the material reported herein has been on strength requirements and not on displacements. As such additional efforts are needed to allow reliable estimation of the increased displacements due to geometric nonlinearities.

12. The above parameter study was aimed at strengthening a structure sufficiently to obtain the same maximum inelastic displacements in systems with and without $P-\Delta$ effects. While the procedures outlined accomplish this, the displacements developed

by the systems may be in many circumstances vastly larger for the system subject to $P-\Delta$ effects. This is because strengthening increases the yield displacement. In some cases these displacements may be excessive and efforts to limit the maximum displacement to a specified value need to be developed as well.

13. A related issue is that the required ductility demand may exceed the design Z value by a considerable margin for large Z , and for structures with periods shorter than the predominant period of the ground motion. In these cases, it may also be worthwhile to investigate procedures which limit ductilities to a factor (like 1.25) times Z for systems with and without $P-\Delta$ effects,
14. This report provides background for those interested in the effects of geometric nonlinearities on single level bridge structures. The studies indicate deficiencies in several of the methods that have been proposed to date, and suggest promising directions for future design. However, the studies presented herein are neither conclusive nor complete. In particular, the effects of uncertainty in ground motion intensity, duration and frequency content, in site conditions and in the structural restoring force characteristics need to be more fully addressed. These data can be analyzed and interpreted to better define relations for the required strengthening Y , for the required factors of safety and for criteria for establishing situations where $P-\Delta$ effects can be safely neglected. Similarly, issues raised in items 10 through 13 above need to be investigated. More detailed response and cost analyses should be carried out on actual bridge projects to assess the relative consequences of increasing stiffness and/or strength to control $P-\Delta$ effects.

Clearly more realistic nonlinear analyses also need to be carried out utilizing refined member idealizations. Because of the gradual deformation hardening exhibited by members, the effects of geometric nonlinearities may not be as severe as expected on the basis of the simplified single degree of freedom systems; alternatively, influences of joint and foundation deformations may increase $P-\Delta$ effects. At the same time issues related to multiple level viaducts should be addressed and simplified design and analysis methods need to be formulated and evaluated.

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